How we perceive the trajectory of an approaching object

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Various equations that describe how observers could recover the trajectory of an approaching object have been put forward. Many are relatively complex formulations that recover the veridical trajectory by scaling retinal cues, such as looming and changing disparity. However, these equations do not seem to describe human perception as observers typically misjudge trajectory angles. Thus, we examine whether a simpler formulation—one that does not predict veridical judgments—may better explain performance. We test the hypothesis that perceived trajectory is based on a speed ratio: the ratio of lateral angular speed to the sum of looming and changing disparity signals. To discriminate between this and alternative proposals, we examined the effect of object size on trajectory perception: The speed ratio hypothesis predicts that perceived trajectory will become less eccentric with increasing object size, while the alternatives predict that perceived trajectory will be independent of object size. Observers performed a trajectory judgment task in which they compared the trajectory direction of two approaching objects, of the same or different size, seen in separate intervals. We estimated perceptually parallel trajectories from their responses. In Experiment 1, objects differed in horizontal and vertical size, and in Experiment 2, they differed only in vertical size. In both experiments, observers’ data showed a clear effect of object size and were close to predictions of the speed ratio hypothesis. We conclude that the alternate proposals we tested were not supported and that the speed ratio account is a sufficient account of the data.

Keywords: motion perception, depth perception, trajectory perception, looming


Introduction

How might we judge the trajectory of an approaching object? The geometry underlying this problem was first described in Hoyle’s (1957) science fiction book, “The black cloud.” In this book, the characters are faced with the problem of determining whether an approaching object will hit the Earth (see Rushton & Gray, 2006). They note that collision can be determined by comparing the rate at which the angular size of the object is increasing to the rate at which its seen position relative to the background stars is changing. Starting with this relative rate of change of angular size and lateral speed, let us derive some of the standard equations for obtaining trajectory direction.

A geometrical basis for perception of trajectory

First, we define our terms. Consider a sphere of width $S$ at a distance $D$ directly in front of the observer and on an oblique trajectory as in Figure 1 (left panel). The sphere is seen by two eyes that are separated by a distance $L$, $X_c$ is the “crossing distance” at which the ball will pass the observer (measured in the frontal plane containing the two eyes). The sphere in the left panel has trajectory angle $\beta$ with respect to the median plane of the head. Lateral visual direction is given by $\alpha$ (illustrated in the right panel). The rate of change of lateral visual direction, or lateral angular speed, is $\dot{\alpha}$. Because we have two eyes, we can define $\alpha$ and $\dot{\alpha}$ for the each eye separately as $\alpha_L$ and $\dot{\alpha}_L$ and $\alpha_R$ and $\dot{\alpha}_R$.

We will call the object’s angular width $\theta$ and its rate of change $\dot{\theta}$. $\theta$ is the angle at the eye subtended by the horizontal extent of the object, and we can define a similar angle, $\phi$, which is the angle at the center of the object that is subtended by the two eyes. $\phi$ is the object’s absolute (or optic array) disparity. This quantity is approximately equal to the retinal disparity, $\alpha_R - \alpha_L$, when disparities are measured relative to a distant point, $P$ (see Sousa, Brenner, & Smeets, 2010). The rate of change of $\phi$ is $\dot{\phi}$.
We will now examine what useful information about trajectories is provided by the variables we defined above. A very simple source of information about an object’s trajectory direction is the ratio of lateral speed to rate of change of angular width (Equation 1). This quantity is unaffected by the object’s physical speed:

\[
\frac{\dot{\alpha}}{\dot{\theta}}.
\]

Equation 1 provides sufficient information to support accurate perceptual discrimination in only certain very limited circumstances. For example, if an observer sees two approaching objects in succession, both of the same size and at the same instantaneous direction, \(\alpha\), then Equation 1 provides sufficient information to determine which will pass furthest from the head. Equation 1 provides ambiguous information about an object’s trajectory. Objects traveling on the same trajectory produce different values when their widths are different, and objects on parallel trajectories produce different values when at different distances from the observer.

As noted, \(\phi\) is geometrically similar to \(\theta\). The similarity is evident from Figure 2. If an object moves toward the observer, angles \(\phi\) and \(\theta\) will both increase and so both provide motion-in-depth information. Indeed, if the

![Figure 1](image1.png)

Figure 1. An object of size \(S\) at distance \(D\) is seen moving in depth along a linear trajectory with angle \(\beta\). The left panel shows the object straight ahead on a trajectory that intersects the plane of the eyes at a distance \(X_c\) from the point midway between the eyes. The object has angular size, \(\theta\), and absolute disparity, \(\phi\); changes to these quantities provide motion-in-depth information. The right panel shows the object further along the trajectory from its earlier point, \(P\), in order to illustrate head-centric visual direction, \(\alpha\), and visual direction at each eye, \(\alpha_L\) and \(\alpha_R\).

![Figure 2](image2.png)

Figure 2. An object on a trajectory approaching the head. Angles \(\theta\) and \(\phi\) both increase as the object approaches. Thus, motion-in-depth information is provided by both \(\theta\) and \(\phi\). These two motion-in-depth signals remain approximately in a constant ratio, \(S/\theta\), independent of distance (see Regan & Beverley, 1979; Regan & Gray, 2000).
object’s width is equal to the observer’s interocular distance, then angles \( \theta \) and \( \phi \) are equal and so are their rates of change, \( \dot{\theta} \) and \( \dot{\phi} \).

We can substitute \( \dot{\phi} \) for \( \dot{\theta} \) in Equation 1 to obtain an alternative cue to trajectory direction:

\[
\dot{\phi} = \frac{c}{\dot{\theta}}.
\]

As noted, we can consider the lateral motion at the left and right eyes separately. Since \( \dot{\phi} = \dot{\alpha}_R - \dot{\alpha}_L \) and \( \dot{\alpha} = (\dot{\alpha}_R + \dot{\alpha}_L)/2 \), we can rewrite Equation 2 as

\[
\frac{\dot{\alpha}_R + \dot{\alpha}_L}{2(\dot{\alpha}_R - \dot{\alpha}_L)},
\]

(see, e.g., Portfors-Yeomans & Regan, 1997). Equations 1–3 provide quantities that vary with trajectory angle, \( \beta \), but they do not support accurate perception and they are not sufficient to guide action. However, scaling these quantities appropriately provides the crossing distance, \( X_c \) (see left panel of Figure 1), which gives the object’s position at some future time, and so is potentially useful for guiding interception or avoiding collisions. For example, Peper, Bootsma, Mestre, and Bakker (1994) proposed that interception could be achieved by comparing the crossing distance of an approaching object with the current position of the hand and executing hand movements to close the gap within the remaining time before arrival; the latter quantity is given by the ratios \( \theta/\dot{\theta} \), \( \phi/\dot{\phi} \) or \( (\theta + \phi)/(\dot{\theta} + \dot{\phi}) \) (See Lee, 1976 and Rushton & Wann, 1999). Thus, visual guidance of action can in principle be achieved without the need for accurate perception of motion direction and speed (see Rushton, 2004 for further discussion).

Crossing distance, \( X_c \), is obtained by scaling Equation 1 by the object’s width, \( S \) (see Bootsma, 1991 for a full derivation):

\[
X_c = \frac{Sa}{\dot{\theta}}.
\]

Thus, \( X_c \) determined in this way expresses the distance that the object will pass the midpoint of the eyes, in units of the object’s width. \( X_c \) can also be obtained by scaling Equations 2 and 3 by \( I \) to obtain

\[
X_c = \frac{I\dot{\alpha}}{\dot{\phi}},
\]

and

\[
X_c = \frac{I(\dot{\alpha}_R + \dot{\alpha}_L)}{2(\dot{\alpha}_R - \dot{\alpha}_L)}.
\]

Note that Equations 4–6 provide the crossing distance in the plane perpendicular to the object’s visual direction at \( D = 0 \). This is the frontal plane at the eyes when the object is straight ahead (Figure 1, left panel). To obtain crossing distance in the plane of the eyes when the object is not straight ahead, the results of Equations 4–6 need to be scaled by \( \cos \theta \).

The object, the midpoint of the eyes, and the crossing distance \( X_c \) define a right-angled triangle. Therefore, we can define \( \beta \) as

\[
\beta = \tan^{-1}\left(\frac{X_c}{D}\right),
\]

and we can substitute Equations 4–6 for \( X_c \) in Equation 7 to obtain the following three formulations for trajectory angle:

\[
\beta \approx \tan^{-1}\left(\frac{S\dot{\alpha}}{D\dot{\phi}}\right),
\]

\[
\beta \approx \tan^{-1}\left(\frac{I\dot{\alpha}}{D\dot{\phi}}\right),
\]

and

\[
\beta \approx \tan^{-1}\left(\frac{I(\dot{\alpha}_R + \dot{\alpha}_L)}{2D(\dot{\alpha}_R - \dot{\alpha}_L)}\right).
\]

Equations 8–10 represent three different ways in which trajectory perception may be achieved. Each implies a different neural mechanism. The motion-in-depth signal in each case is looming (Equation 8), changing disparity (Equation 9), and interocular velocity difference (Equation 10). Equation 8 is a formulation for trajectory angle that follows from Equation 4 for crossing distance proposed by Bootsma (1991) and Regan and Kaushal (1994). Equation 9 is a formulation proposed by Regan (1993). Equation 10 is a formulation proposed by Regan (1993) and Portfors-Yeomans and Regan (1997).

Other formulations have also been proposed, for example, Welchman, Tuck, and Harris (2004) showed that it is possible to determine the trajectory angle without knowledge of object size or distance using unscaled retinal information, as in Equation 11; \( \theta_0 \) denotes initial angular size and \( \theta \) is current angular size:

\[
\beta \approx \tan^{-1}\left(\frac{\theta_0\sin\alpha}{\theta - \theta_0\cos\alpha}\right).
\]
Note that while the above formulations are typically described in relation to simple stimuli, this is simply a matter of exposition as they also apply to more complex stimuli. For example, $\theta$ (looming) requires only a pair of points on an object so it is often described in relation to angular width, but it could be estimated from more complex structured or textured objects that provide more data points. Rich stimuli typically evoke strong sensations of motion in depth, so stimulus complexity in this sense does not hinder perception and thus make them less relevant to the equations we present. Displays containing radial motion elicit strong motion aftereffects—stronger than for equivalent lateral motions—suggesting that local motion signals are integrated to produce a looming signal (Bex, Metha, & Makous, 1999).

**Empirical studies: Non-veridical perception of trajectory**

The above formulations (Equations 8–11) in principle allow accurate recovery of direction of motion in depth; however, they do not describe or predict the perception of trajectory by human observers. A wealth of psychophysical findings suggests that observers do not perceive direction of motion in depth accurately. For example, it is commonly found that trajectory angles, such $\beta$ in Figure 1, are overestimated: Approaching objects appear to pass further from the head (e.g., Harris & Dean, 2003; Peper et al., 1994; Welchman et al., 2004). It may be that observers attempt veridical recovery of motion in depth, e.g., by implementing a scheme such as those above, but fail due to incorrectly estimating the necessary parameters, or that they do not attempt veridical recovery.

Harris and Drga (2005) argued that observers do not attempt veridical recovery of motion in depth. In their study, the observers’ task was to match the orientation of a pointer located on the desk in front of them to appear equal to the trajectory angle of a small (8.3 arcmin) approaching binocular target. They found highly inaccurate perception of trajectory angle and argued that judgments are not based on the use of scaled binocular cues that enable veridical recovery of 3D motion, such as described above. They concluded that instead observers use a simple strategy based on the total change in visual direction (Equation 12, where $\beta'$ is perceived trajectory angle):

$$\beta' \propto \Delta a.$$  \hspace{1cm} (12)

Their results suggest that observers did not use binocular cues to judge trajectory angle, even though observers were able to discriminate the same extents in depth displayed statically. In contrast, other studies have reported evidence that observers can use binocular cues to judge trajectory angles. Lages (2006) found that observers used binocular cues in judging the trajectory angle of a small, stereoscopic target; however, the perceived extent of motion in depth was less than veridical. Welchman, Lam, and Bülthoff (2008) found that binocular cues are used in trajectory perception, though they are relatively less effective than lateral motion cues, and this difference could explain why trajectory angles are often overestimated.

We also recently reported (Rushton & Duke, 2007) evidence suggesting that observers do not veridically perceive direction of motion in depth. We examined observers’ ability to “factor out” viewing geometry in perception of trajectory angle. We used an apparently parallel trajectory task in which observers judged the relative orientation of pairs of trajectories at different starting distances and directions with respect to the head. Accurate performance on this task requires accurate interpretation of visual cues to motion in depth; however, performance was very poor and suggested no attempt to scale visual cues. Instead, performance was more consistent with the use of unscaled cues, such as the ratio of lateral angular velocity to looming rate (Equation 1) or the ratio of angular velocity to changing disparity (Equation 2).

Our previous findings provide the starting point for the work we report here. We sought to test our hypothesis that perceived trajectory is based on a ratio of unscaled retinal cues. We start by examining the basis for this hypothesis.

**Early cue combination**

Regan and Beverley (1979) proposed that motion-in-depth perception is based on the early summation of looming and changing disparity signals (i.e., $\theta + \phi$), as opposed to later combination of separate trajectory estimates derived from each of these two cues. (The term “early” cue combination is used here in the original sense used by Regan and Beverley to refer to the combination of unscaled looming and changing disparity signals. This is in contrast to models where cues are combined “late” after scaling for viewing distance.) They note that the magnitude of the looming cue and the magnitude of the changing disparity cue are related by

$$\frac{\dot{\theta}}{\dot{\phi}} \approx \frac{S}{T}. \hspace{1cm} (13)$$

Thus, the relative strength of looming rate and changing disparity varies in relation to the size of the object. This equation illustrates that looming provides a more effective or “useful” cue than changing disparity when the object is larger than the interocular distance. The reverse is true when the object is smaller than the interocular distance. Following Regan and Beverley (1979), the term “usefulness” is used to refer to the relative magnitude of looming and changing disparity signals as in Equation 13.

If looming and changing disparity cues are summed, the combined motion-in-depth signal will be based mostly on the looming cue for objects larger than the interocular
distance and mostly on changing disparity for smaller objects. Thus, early summation naturally weights looming and changing disparity cues in proportion to the usefulness of each cue. This optimal weighting principle is used in Rushton and Wann’s (1999) dipole model of perception of time to contact. A notable feature of this form of cue combination is that it does not involve explicit weights for each cue; there is no need to assume that an observer has prior knowledge of cue reliabilities or “ancillary cues” (see Landy, Maloney, Johnston, & Young, 1995).

Evidence that shows that trajectory can be discriminated on the basis of the ratio of changing visual direction, \( \alpha \), to changing disparity, \( \phi \) (e.g., Portfors-Yeomans & Regan, 1996), or the ratio of changing visual direction to looming rate, \( \theta \) (e.g., Regan & Kaushal, 1994) has been presented. Here, we bring these proposals together to examine whether perception of trajectory angle involves the use of looming and changing disparity signals that are summed at an early stage, consistent with the proposal of Regan and Beverley (1979). This scheme is described by the following equation:

\[
\beta' \propto \frac{\alpha}{\theta + \phi}.
\] (14)

It is evident that Equation 14 does not predict veridical perception. Looming and changing disparity signals are simply added together, and they are not scaled by an estimate of the target’s distance. The idea that trajectory perception might be explained by a simple non-veridical strategy was also examined by Harris and Draga (2005) (see equation 12 above). Let us consider the consequences of Equation 14 for trajectory perception.

**Trajectory perception and object size**

If trajectory angle perception is based on the ratio of changing visual direction to the sum of looming rate and changing disparity, as in Equation 14, then observers should perceive smaller objects traveling on a trajectory as in Figure 1 as having a larger trajectory angle (i.e., larger crossing distance). In contrast, the formulations outlined earlier (Equations 8–12) do not predict an effect of object size on trajectory perception.

There is some evidence in the existing literature for an effect of size on perception of trajectory direction. Using indirect response methods, Jacobs and Michaels (2006), Peper et al. (1994), and Welchman et al. (2004) found evidence that smaller objects appear to have larger trajectory angles than larger objects, i.e., smaller objects appear to pass further from the head.

Here, we test this prediction directly. We examine trajectory perception using differently sized objects to determine whether the early cue combination model may explain performance. We do so by using an apparently parallel trajectory task (see Rushton & Duke, 2007) in which observers see pairs of objects approaching on different trajectories presented sequentially. Observers judged whether the trajectory in the second interval was moving more leftward or rightward than the first. The task allows us to determine which physical trajectories are perceptually parallel (50% thresholds in our discrimination task), and deviations from physically parallel indicate the magnitude of the effect of our experimental manipulations.

In Experiment 1, we examine the effect of object size and assess the extent to which perceptual errors can be explained by the use of early combination of looming and changing disparity cues (Equation 14). We further test this hypothesis in Experiment 2 by using objects of different shape.

### Experiment 1

#### Methods

**Observers**

Four observers took part. All had normal or corrected vision and stereoaucity of at least 70 arcsec measured with the Stereoptical Randot Stereotest. All were experienced psychophysical observers. Observer PD was an author. All others were paid naive volunteers.

**Apparatus and stimuli**

Stimuli were stereoscopic images generated on a PC computer and viewed using Stereographics CrystalEyes LCD shutter goggles synchronized with a 22” Viewsonic p225fl Flat Screen CRT at a refresh rate of 100 Hz, i.e., alternating left and right eye images were displayed at a refresh rate of 50 Hz per eye. Images were rendered using the OpenGL Graphics Library. Stereoscopic image projection was geometrically correct, which provided accurate rendering of perspective. It also ensured that both horizontal and vertical disparities were correct and naturally in agreement, which is important since vertical disparities are used to scale horizontal disparities in stereopsis (Howard & Rogers, 2002). The luminance profile of the display was linearized and anti-aliasing techniques were used to minimize pixelation. Stimuli were displayed in red as this gave the most effective separation of the left and right eyes’ images. A red acetate filter placed in front of the CRT screen was used to enhance the black level of the monitor. These factors made for a clear, vivid stereoscopic display that promoted a strong sensation of motion in depth. Observers viewed the display with the head held still in a head and chin rest at a fixed distance of 1.5 m from the display. The display was viewed in a dark laboratory in which nothing other than the stimulus display was visible.

The stimuli comprised either a large (5.2-cm diameter) or small (2.4 cm) sphere that approached the observer on a
linear trajectory in the presence of a static surround. The surround consisted of six large squares located in the plane of the display screen at 1.5 m (see Figure 3). These large squares provided a very easily fusible reference plane and were designed to provide a powerful relative disparity cue between the surround and the spheres. The spheres (solid wire-frame spheres comprising 15 lines of longitude and 15 lines of latitude) approached the observer on linear trajectories in the horizontal plane containing the nodal points of the eyes.

At the start of its trajectory, the sphere was located directly in front of the observer at a distance of 1.75 m (i.e., 25 cm beyond the CRT surface). It was seen stationary in this position for 0.5 s to provide sufficient time for observers to fuse the stimulus. After this time, the sphere traveled along a linear trajectory, which would pass either to the left or right of the head. The spheres traveled at 90 cm/s ($T_{20\%}$) and were seen moving for 0.7 s ($T_{20\%}$).

The speed and duration of each trial was varied randomly in order to discourage the use of incidental cues, such as displacement in the image plane. Spheres disappeared at a distance of 1.12 m on average. We describe the trajectories in terms of their crossing distance $X_c$, i.e., the lateral distance at which the sphere would intersect the frontal plane at the eyes. We also describe the trajectory in terms of its angle $\beta$ from the median plane (Figure 1, left panel), and we provide both quantities on our graphs in the Results and analysis section.

**Procedure and task**

On each experimental trial, the observer was shown a pair of trajectories, seen sequentially. One of these was a “reference trajectory” with a crossing distance of either 1, 2, 4, or 8 cm to the left or right of the midline (i.e., trajectory angles of 0.3, 0.7, 1.3, and 2.6$^\circ$). The other was a “test trajectory” whose angle was determined by a simple staircase procedure. The order of reference and test trajectory within the pair was random. The observers’ task was to indicate via key press whether the trajectory in the second interval was moving more leftward or rightward than the first. The staircase converged at the PSE and then sampled a range of angles around that point (see Rushton & Duke, 2007). Each staircase comprised data from 80 trials. Probit analysis of these data produced an estimate of observers’ apparently parallel trajectory and their 75% discrimination thresholds in each condition. Data were collected in the following conditions:

**Same size**—either both reference and test spheres were large, or both were small. We would not expect any systematic bias in trajectory judgments in these conditions. We included these conditions to examine the influence of size on 3D trajectory discrimination thresholds.

**Different size**—either the reference trajectory sphere was large and the test trajectory sphere was small (large reference, small test) or vice versa (small reference, large test). Using these data, we can examine the influence of object size on visual judgments of apparently parallel 3D trajectories.

Same-size and different-size conditions were presented in separate sessions, each comprising 640 trials and each lasting approximately 45 min.

**Results and analysis**

Results for the different-size conditions are shown in Figure 4. This figure shows apparently parallel trajectory data and discrimination threshold data in the different-size conditions. Results for the same-size conditions are described, as these conditions are only included to assess discrimination thresholds, but they are shown in Supplementary Figure S1.

**Trajectory discrimination thresholds**

In the same-size conditions, discrimination thresholds were very similar in the small- and large-sphere conditions (on average, 0.6 cm and 0.4 cm, respectively) but were affected more by crossing distance. Observers were more sensitive to changes in trajectory nearer the midline. Thresholds increased with increasing crossing distance (on average, from 0.2 cm to 1.0 cm over the 1–8 cm range of crossing distances tested). A 2-factor ANOVA (sphere size, crossing distance) performed on the discrimination threshold data for the same-size conditions revealed a main effect of crossing distance ($F_{3,9} = 10.522, p = 0.003, \eta^2 = 0.52$) and no effect of the size of the spheres alone or in interaction with crossing distance. These data are provided in Supplementary Figure S1.

**Figure 3.** Schematic diagram of the stimulus display. Observers saw red wire-frame spheres approaching on linear trajectories. Spheres were seen against a static background. Stimuli were seen in darkness. See text for details.
In the different-size conditions, discrimination thresholds were similar in the small-reference, large-test and large-reference, small-test conditions (on average, 1.0 cm and 0.4 cm, respectively). As with the same-size conditions, thresholds increased slightly with increasing crossing distance (on average, from 0.3 cm to 1.2 cm). These differences were small, and a 2-factor ANOVA (sphere size, crossing distance) revealed no significant effects of...
size or crossing distance alone or in interaction. Discrimination threshold data for the different-size conditions are shown as error bars in Figure 4.

Discrimination thresholds in the different-size conditions were similar to those in the same-size conditions. To compare the two, we calculated, for each reference trajectory crossing distance, each observer’s average threshold in the same-size and different-size conditions. A two-factor ANOVA (same-size vs. different-size, reference trajectory crossing distance) revealed a significant effect of crossing distance ($F_{3,9} = 6.395$, $p = 0.013$, $\eta^2 = 0.50$), as found in the same-size conditions, but no effect of same- vs. different-size condition alone or in interaction with crossing distance.

Overall, the discrimination threshold results demonstrate that both the large- and small-sphere stimuli produce reliable performance. Observers were able to reliably discriminate between trajectories that differ in crossing distance by around 1 cm, with no difference in performance between the small or large spheres. The finding of greater sensitivity for trajectories closer to the midline replicates previous findings (see Beverley & Regan, 1975; Portfors-Yeomans & Regan, 1996) and is expected since the relevant visual cues ($\alpha$, $\theta$, and $\phi$) change progressively less for each unit increase in crossing distance.

**Apparently parallel trajectory estimates**

Apparently parallel trajectory estimates in the different-size conditions are shown in Figure 4. All observers’ results in the different-size conditions show the same pattern of systematic error. Observers perceived the test and reference trajectories as perceptually parallel when the small sphere had a physically smaller crossing distance than the large sphere. That is when large and small spheres move along the same physical trajectory, the small spheres were seen to have larger crossing distances than large spheres. On average, trajectories appeared parallel when the crossing distance of the small-test trajectory was 71% of the large-reference crossing distance and when the largest-test crossing distance was 147% of the small-reference crossing distance. Overall, without regard for whether a given trajectory was a reference or a test, data indicate that trajectories appeared parallel when the crossing distance of the small spheres was 69% of that for the large spheres.

A 2-factor ANOVA (sphere size, crossing distance) performed on the apparently parallel trajectory data from the different-size conditions (Figure 5) revealed a significant effect of size ($F_{1,3} = 62.351$, $p = 0.004$, $\eta^2 = 0.15$), crossing distance ($F_{3,9} = 575.578$, $p < 0.0005$, $\eta^2 = 0.74$), and size in interaction with crossing distance ($F_{3,9} = 28.483$, $p < 0.0005$, $\eta^2 = 0.09$).

For completeness, data for the apparently parallel trajectory estimates in the same-size conditions are shown in Supplementary Figure S1 for each observer (a–d) and the group mean (e). These data show no systematic bias in any condition, as expected.

**Analysis of apparently parallel trajectory estimates**

Here, we examine whether the systematic pattern of trajectory perception errors found in the different-size conditions of Experiment 1 (Figure 4) can be explained by observers using a motion-in-depth signal given by the sum of looming and changing disparity signals (Equation 14). As described in the Early cue combination section, summation of these signals naturally “weights” each signal according to its strength (i.e., its usefulness; see Regan & Beverley, 1979; Rushton & Wann, 1999). In our analysis, we can examine whether these two signals are combined in these expected proportions. To do so, in our analysis, we include parameter $g$ that determines the relative gain for looming rate vs. changing disparity (see Equation 15). The two signals are combined in the expected proportions when $g = 0.5$:

$$
\beta' \sim \frac{\alpha}{g\theta + (1-g)\phi}.
$$

(15)

Note that a gain of 0.5 does not mean that looming and changing disparity contribute equally to the motion-in-depth signal. It means that the two signals are simply added together without any additional bias toward the use of the looming signal ($g > 0.5$) or the changing disparity signal ($g < 0.5$).

In this model, test and reference trajectory angles will appear parallel (i.e., $\beta'_{\text{test}} = \beta'_{\text{reference}}$) when

$$
\frac{a_{\text{test}}}{g\theta_{\text{test}} + (1-g)\phi_{\text{test}}} = \frac{a_{\text{reference}}}{g\theta_{\text{reference}} + (1-g)\phi_{\text{reference}}}.
$$

(16)

For a given value of $g$, we can determine the physical test trajectory angle that would appear equal to a physical reference trajectory under this model, i.e., we find the physical test stimulus trajectory angle that satisfies Equation 16. We do so using a numerical method. Predictions of this model are given as follows:

1. If observers made no use of changing disparity, and instead based their perception of trajectory angle on $a/\theta$, then $g = 1$. In this case, expected performance on the apparently parallel judgment task is shown as fine dashed lines in Figure 4. The slopes of these lines are equal to the ratios of the physical size of the test and reference spheres.

2. If observers made no use of looming signals, then $g = 0$. In this case, the size of the spheres would not influence judgments, so there will be no systematic error. Expected performance is shown as the diagonal line, $y = x$. Data lying on this line would tell us that observers do not use looming signals, but such data would not be informative about the extent to which observers use changing disparity signals (data on this line indicate that physically identical...
Figure 5. Results from Experiment 2 shown in the same format as those for Experiment 1. The solid red circles show data from the small-sphere-reference, tall-ellipsoid-test object conditions and open blue circles show data from the tall-ellipsoid-reference, small-sphere-test conditions. For each of the two conditions, fine dashed lines indicate theoretical performance if observers perform the task using $g = 1$, i.e., gain $g$ on $\hat{\theta}$ is equal to 1 in Equation 16. In this case, motion-in-depth perception is based only on looming, not changing disparity. Note that $\hat{\theta}$ for test and reference objects are more similar in this experiment than Experiment 1 because the two types of object differed in size only in the vertical direction ($\hat{\theta}$ is calculated as the average change in image size over the horizontal and vertical dimensions); consequently, theoretical performance for strategies using $\hat{\theta}$ lie closer to $y = x$. Bold dashed lines show performance if looming and changing disparity signals are summed to give a motion-in-depth signal ($g = 0.5$, i.e., an equal gain on looming and changing disparity signals; thus, the signals are unweighted). The line $y = x$ indicates test trajectories that are physically identical to the reference trajectories. This line indicates no effect of object size, i.e., the gain on looming is zero. Fine solid lines show best fitting model data. Error bars show 75% discrimination thresholds in (a)–(d) and the group mean thresholds in (e). All observers perceived the small spheres as traveling on trajectories passing further from the midline of the head than the tall ellipsoids. For all observers, performance was close to that expected if trajectory perception uses a motion-in-depth signal that is based on the sum of looming and changing disparity signals, with an equal gain on each (bold dashed lines).
test and reference trajectories appear identical, but it does not mean perception is veridical because both may be misperceived by the same amount).

3. If observers used the sum of looming and changing disparity, consistent with the proposal of Regan and Beverley (1979), then $g = 0.5$. Expected performance in this case is shown as bold dashed lines.

Figure 4 shows that for all observers, apparently parallel trajectory settings lie close to the expected settings based on the sum of looming and changing disparity. Best fitting (least-squares) model data were obtained for each participant’s data. For each participant, the model was fit to observed data from the small-reference, large-test (Figures 4a–4e, red circles) and large-reference, small-test (blue circles) conditions separately. Gain values from this analysis are shown in Table 1. These values are reasonably similar across observers (standard deviations were 0.08 in the small-reference, large-test conditions and 0.16 in the large-reference, small-test conditions). In almost all cases, observers showed a slight deviation, corresponding to relatively greater use of looming than predicted, i.e., $g > 0.5$. Best fitting data for the group mean corresponded to a gain on looming of 0.64 in the small-reference, large-test condition and 0.51 in the large-reference, small-test condition. A paired-samples $t$ test revealed no significant difference. The average gain value across the two conditions was 0.58, indicating a slightly greater contribution of looming than changing disparity (0.42). As stated earlier, the crossing distance of the small spheres was found to be 69% of that for the large spheres. If looming and changing disparity signals had been summed with equal gains (i.e., $g = 0.5$), the crossing distance of the small spheres would have been 76% of that for the large spheres (the slope of the bold dashed light blue line in Figure 4). This prediction is close to the observed value.

### Discussion

Experiment 1 revealed very little influence of object size on observers’ sensitivity to trajectory angle, but importantly, there was a large influence of object size on bias in trajectory perception. Smaller objects appeared to have larger trajectory angles than larger (i.e., wider and taller) objects. The two appeared to have the same trajectory angle when the crossing distance of the small object was 69% of that for the large object. The direction of this result is consistent with the results of previous studies in which the effect of object size on trajectory perception was examined (Jacobs & Michaels, 2006; Peper et al., 1994; Welchman et al., 2004). We note that similar systematic biases have been found previously using both computer-simulated objects and real objects such as differently sized balls and LEDs (Peper et al., 1994; Welchman et al., 2004). This suggests that our results are not simply the result of cue conflict between accommodation and vergence/disparity in the displays, which can influence perceived depth (Watt, Akeley, Ernst, & Banks, 2005).

How might the effect of size be explained? An effect of object size on performance is not predicted by the formulations outlined in the Introduction section (Equations 8–12) with the exception of the early cue combination theory (Equation 14). Under this theory, looming and changing disparity cues are simply added, without weighting, to give a motion-in-depth signal that is used in trajectory perception. Figure 4 shows that observers’ apparently parallel trajectory settings were consistently close to the values predicted by this theory. Thus, the data support the conclusion that trajectory perception is based on a motion-in-depth signal derived from early summation of looming and changing disparity signals.

We found that all observers showed the same general pattern of results, and analysis of the group mean data produced gain values for looming vs. changing disparity cues that indicate that the cues are combined in proportions close to those expected based on their usefulness (Regan & Beverley, 1979). However, the analysis indicated some individual differences in the effectiveness of each cue (Table 1). This finding is consistent with a recent study by Nefs, O’Hare, and Harris (2010) examining individual differences in the relative contribution of different motion-in-depth signals. That study revealed individual differences in the contribution of changing disparity vs. interocular velocity difference in motion-in-depth perception. Observers who responded more to changing disparity responded less to interocular velocity difference. An individual’s relative use of each cue in that study, and the present study, may be related to their binocular disparity sensitivity.

### Experiment 2

In Experiment 1, we found evidence to support the hypothesis that trajectory perception is based on early summation of looming and changing disparity signals.
However, an alternative explanation for the effect of size can be proposed based on the works by Peper et al. (1994) and Regan and Kaushal (1994). Considering trajectories in the horizontal meridian plane, these studies note a potential role for the ratio of changing horizontal visual direction to changing horizontal optical size. This ratio specifies a trajectory’s horizontal crossing distance, \( n \), in units of the object’s horizontal physical size and thus could provide a way to encode trajectories without involving distance information:

\[
\frac{\hat{\alpha}}{\hat{\theta}} = n
\]  

(17)

Such a scheme could explain the finding that small balls appear to pass further from the head if trajectories are encoded in terms of \( n \), i.e., without scaling this quantity by object width. If observers perceive trajectories within the horizontal meridian plane in terms of crossing distance in units of horizontal physical size, as in Equation 17, then perceived trajectory should not depend on the object’s vertical size. We tested this in Experiment 2. Observers compared the trajectories of a small sphere and a “tall” ellipsoid whose horizontal size was the same as the small sphere but whose vertical size was larger. Unlike a sphere, the ellipsoid stimulus provides a different looming signal on its horizontal and vertical axes. If only the horizontal size is important, then the vertical size manipulation should have no effect. Alternatively, if both dimensions are important, then apparently parallel trajectory judgments should reflect the use of a looming signal derived from retinal velocities measured in both horizontal and vertical directions. The latter prediction is supported by evidence from Gray and Regan (2000). They used stimuli with unequal rates of horizontal and vertical expansion in the retinal image and found that time-to-contact judgments, in monocular conditions, were influenced by the velocities in both horizontal and vertical directions.

**Methods**

**Observers**

Participants in Experiment 2 were the same four participants from Experiment 1.

**Apparatus and stimuli**

The apparatus for Experiment 2 was identical to that for Experiment 1. The stimuli included the same small sphere as in Experiment 1 (size = 2.4 cm), but the large sphere was replaced with a tall ellipsoid. The horizontal size of the ellipsoid was the same as the small sphere (2.4 cm) and its vertical size was the same as the large sphere in Experiment 1 (5.2 cm). All other aspects of the stimuli were identical to Experiment 1.

**Procedure and task**

The procedure was the same as that for Experiment 1 except observers compared the trajectories of the small sphere and the tall ellipsoid (they did not also compare the same object types unlike Experiment 1). Each observer performed 640 trials that lasted approximately 45 min.

**Results and analysis**

The results of Experiment 2 are shown in Figure 5. This figure shows apparently parallel trajectory estimates for each observer (Figures 5a–5d) and the group mean (Figure 5e).

**Apparently parallel trajectory estimates**

As in Experiment 1, all observers’ results were similar. Figure 5 shows that when small spheres and tall ellipsoids were on apparently parallel trajectories, the small sphere had a smaller crossing distance than the tall ellipsoid. That is the small spheres were seen to have larger crossing distances than the tall ellipsoids when the two were on the same physical trajectory. When the test stimulus was a small sphere, the crossing distance of the estimated apparently parallel trajectory was 80% of the tall-ellipsoid-reference crossing distance, on average. When the test stimulus was the tall ellipsoid, its crossing distance was 121% of the small reference. Overall, without regard for whether a given trajectory was a reference or a test, the crossing distance of the small spheres was 81% of that for the tall ellipsoids. A 2-factor ANOVA (shape, crossing distance) performed on the data of Figure 5 revealed a significant effect of vertical size \( F_{1,3} = 11.698, \ p < 0.042, \ \eta^2 = 0.08 \), crossing distance \( F_{3,9} = 214.997, \ p < 0.0005, \ \eta^2 = 0.85 \), and vertical size in interaction with crossing distance \( F_{3,9} = 28.483, \ p < 0.0005, \ \eta^2 = 0.04 \).

**Trajectory discrimination thresholds**

Discrimination thresholds were very similar in the small-sphere-reference, tall-ellipsoid-test and tall-ellipsoid-reference, small-sphere-test conditions (on average, 0.5 cm and 0.3 cm, respectively) but were affected more by crossing distance. Thresholds increased with increasing crossing distance by a small amount (on average, from 0.2 cm in the 1-cm crossing distance condition to 0.6 cm in the 8-cm condition). A 2-factor ANOVA (vertical size, crossing distance) performed on the discrimination threshold data of Experiment 2 revealed a significant effect of crossing distance \( F_{3,9} = 5.035, \ p = 0.026, \ \eta^2 = 0.26 \) and no effect of the object’s vertical size alone or in interaction with crossing distance.

**Analysis of apparently parallel trajectory estimates**

To examine whether observers’ apparently parallel trajectory judgments could be explained by summation...
of looming and changing disparity signals as previously described, data from Experiment 2 were analyzed in the same way as in Experiment 1. Values for $\theta$ used in this analysis were the average of the value in the horizontal and vertical dimensions. Thus, we compared observers’ judgments against a model that made the simplest assumption about the use of looming information in horizontal and vertical directions—that velocities in horizontal and vertical directions are weighted equally in producing the looming signal. It is possible that the visual system uses a more complicated weighting rule and further studies are needed to examine this matter in detail. Nevertheless, evidence suggests that retinal velocities are combined across different directions in the image (Gray & Regan, 2000). As per the analysis of Experiment 1, we plot predicted apparently parallel trajectory values as follows: (1) where $g = 0$, the motion-in-depth signal used in trajectory perception is not derived from vertical changing image size information; (2) where $g = 1$, the motion-in-depth signal is derived from looming signals (where horizontal and vertical changing sizes contribute equally); and (3) where $g = 0.5$, motion in depth is based on the sum of looming and changing disparity, consistent with the proposal of Regan and Beverley (1979).

The best fitting apparently parallel trajectory estimates generated by the model are shown in Figure 5 as fine solid lines. The gain parameter values corresponding to these data are given in Table 2. These values are generally similar across observers but more variable than in Experiment 1 (standard deviation in the small-sphere-reference, tall-ellipsoid-test conditions was 0.26 and 0.16 in the tall-ellipsoid-reference, small-sphere-test conditions). Greater variability is expected in Experiment 2 since the differences in between the model’s predictions for $g = 0$ and $g = 1$ are smaller than in Experiment 1, and therefore, parameter estimates will be more susceptible to noise.

Best fitting data for the group mean corresponded to a gain of 0.70 in the small-sphere-reference, tall-ellipsoid-test condition and 0.53 in the tall-ellipsoid-reference, small-sphere-test condition (a paired-samples $t$ test indicated no significant difference). The average gain value for looming was 0.61, indicating a greater contribution of looming than changing disparity (i.e., 0.39). This bias was larger than in Experiment 1 (looming gain was 0.54). Almost all observers in both of the reference-type conditions showed a deviation, corresponding to relatively greater use of looming than predicted, i.e., $g > 0.5$, as in Experiment 1.

If looming and changing disparity signals had been summed with equal gains on each signal (i.e., $g = 0.5$), the crossing distance of the small spheres would have been 86% of that for the tall ellipsoids (the slope of the bold dashed blue line in Figure 5). This prediction is close to the observed value of 81%.

### Discussion

Experiment 2 revealed similar trajectory discrimination performance to Experiment 1 and we still found large systematic errors in apparently parallel judgments despite the two objects differing only in vertical size. Thus, we can conclude that observers make use of vertical changing image size in judging the trajectory of objects approaching in the horizontal meridian plane. The magnitude of the effect of vertical size on apparently parallel trajectory judgments was close to the magnitude predicted by the use of a motion-in-depth signal obtained from the sum of a looming signal (which is derived from the 2-dimensional image) and changing disparity signals. The predicted magnitudes are shown by the bold dashed lines in Figure 5.

The finding of an influence of size—this time in the vertical direction only—is not predicted by any of the equations outlined in the Introduction section. Nor is it predicted if trajectory is encoded as the crossing distance expressed as multiples of the object’s width (Peper et al., 1994; Regan & Kaushal, 1994), or if observers use a strategy based on the judged direction of the nearest horizontal edge of the objects. However, the result is predicted by the early cue combination hypothesis. Experiment 2 found an effect of object size close to predicted values; thus, it adds further support to the early cue combination theory as an account of trajectory perception.

Experiment 2 also provides evidence that observers did not use a strategy based on judging the direction of the nearest edge of the spheres in Experiment 1. Note that if the observer were to judge trajectories on the basis of the head-centric direction of the nearest horizontal edge at the point of disappearance, a systematic bias would result when the objects are of different widths. A small test object would be set to a smaller trajectory angle than a large reference object, as we have found. This possibility is unlikely given that we randomized the disappearance point.
of the trajectories; however, Experiment 2 provides a way of ruling this out definitively as no such bias could occur when test and reference objects have the same width.

General discussion

The present experiments and several previous studies have shown that the size of an approaching object influences the perception of its trajectory (Jacobs & Michaels, 2006; Peper et al., 1994; Welchman et al., 2004). This result is not predicted by theories of trajectory perception that propose that retinal cues (such as retinal velocity, looming, changing disparity, and interocular velocity difference) are scaled using distance information to recover trajectory angle. A further possibility may be that observers attempt accurate distance scaling of these retinal cues but do so in error due to misestimating the distance to the objects. Under this hypothesis, observers overestimate the distance of small objects relative to large ones (Collett, Schwarz, & Sobel, 1991; Sousa, Brenner, & Smeets, 2011), and thus, the trajectory angle of a small object would be underestimated relative to that of a large object on the same physical trajectory. Consequently, for a small and a large object to appear on parallel trajectories, the physical trajectory angles of the small objects would need to be greater than those of the large objects. This is the reverse of what we found and we can, therefore, reject this hypothesis (see Appendix A). In addition, the effect of size on trajectory perception is not predicted by the theory of Harris and Drga (2005) that trajectory perception is based on the total change in azimuth of an approaching object.

Instead, we examined whether the effect of object size on trajectory perception could be explained by the visual system using a motion-in-depth signal that is the sum of looming and changing disparity signals. This early combination of motion-in-depth signals was suggested by Regan and Beverley (1979), and here, we examined it in the context of an apparently parallel trajectory judgment task for objects approaching in the horizontal meridian plane. This theory offers a simple account of trajectory perception errors that arise due to differences in object size. In this theory, the summation of looming and changing disparity signals is a simple form of cue combination in which the resulting motion-in-depth signal is based more on the stronger (and therefore most useful) of the looming and changing disparity signals. When applied to perception of trajectories, this theory predicts that smaller objects will be seen to have larger trajectory angles (larger crossing distances) than larger ones, and therefore, it offers a potential explanation of the observed effect of object size.

Under this theory, the visual system does not achieve veridical trajectory perception. This is a disadvantage, but it comes with the benefit of reduced computational complexity, which could favor fast processing of dynamic stimuli, and the resulting percept would be sufficient for visually guided action, e.g., reaching for an approaching object. Indeed in terms of evolutionary pressures, a system that supports action would be more important than one that supports accurate perception of 3D motions in terms of angles and extents. Further, a bias toward judging larger objects as passing closer to the head than smaller ones might confer an ecological advantage in terms of avoiding dangerous collisions. While the present findings highlight that trajectory perception is inaccurate, it is still sufficient to support action, e.g., intercepting an object such as catching a ball (Peper et al., 1994) or playing table tennis (Bootsma & van Wieringen, 1990). Although predictive spatial information such as crossing distance is perceived inaccurately, its accuracy is improved by the use of knowledge about the object’s size, and this can lead to improved manual judgments (Peper et al., 1994). López-Moliner, Field, and Wann (2007) showed the potential importance of object size knowledge in simplifying the computations involved in interceptive action. Further, adaptation within the motor system serves to minimize errors in action performance through the use of perceptual feedback. Judge and Bradford (1988) demonstrated that while subjects are initially unable to catch a ball under telestereoscopic viewing conditions, normal catching accuracy is rapidly restored after a few trials. Importantly, accurate actions can be achieved through the continuous coupling between perceptual information and motor commands. Peper et al. (1994) provided evidence of a coupling between hand velocity and perceptually specified required velocity, which provides a means of getting the hand to the target regardless of the perceived location of the target. In this way, accurate interaction with a dynamic visual world could be achieved even though visual perception of 3D motion direction and speed is typically inaccurate (e.g., Lages, 2006; Lugtigheid, Brenner, & Welchman, 2011; Rushton & Duke, 2007, 2009; Welchman et al., 2004).

The present study shows that early combination of looming and changing disparity signals is sufficient to account for the effect of object size on trajectory perception; however, a higher level account is also possible. The effect of size on trajectory perception may be explained by cue combination at the level of 3D trajectory estimates. Such a scheme could involve weighted averaging of trajectory angle estimates derived separately from looming and from disparity cues (e.g., Ernst & Banks, 2002; Landy et al., 1995). Welchman et al. (2004) suggested that it is possible to explain the effect of size if the two estimates disagree and if the relative weighting of the two estimates varies with object size. According to this account, smaller balls appear to pass further from the head because observers
have a biased estimate of trajectory derived from disparity/vergence cues, which is combined with a less biased estimate of trajectory from looming. If the estimate of trajectory angle from looming is made less reliable by reducing the object’s size, then the estimate of trajectory angle will become based more on changing disparity. Thus, perceived trajectory angle will become larger as the object size is reduced. Why might the trajectory estimates derived from looming and from changing disparity disagree? Disagreements may arise for many reasons, for example, due to an error in the distance estimate used in scaling disparity information. This may be particularly likely in computer displays, but even when viewing real objects, accurate 3D perception is the exception rather than the norm (e.g., Porrill et al., 2010; Wagner, 1985; Welchman et al., 2004). While the present study is unable to differentiate between the early and higher level cue combination accounts, it makes the point that a higher level account is not necessary since trajectory perception errors can be explained by our simple model.

In both of the present experiments, using differently sized and shaped objects, we found that our data could be explained by early cue combination of looming and changing disparity signals. In both cases, the results of our analyses indicated that the gains on looming and changing disparity signals were similar, which would be expected if the two signals are simply summed without weighting. As mentioned in the Introduction section, this scheme naturally combines looming and changing disparity cues in proportion to their usefulness.

This study shows that it is possible to explain the effect of size on trajectory perception in terms of a very simple early visual process and does not require an explanation in terms of visual processes at a higher level, such as combination of 3D trajectory estimates from multiple cues.

**Summary**

This study examined the effect of object size on trajectory perception using an apparently parallel trajectory task. Observers overestimated the trajectory angle of smaller objects relative to larger ones, consistent with previous reports. This pattern of results is inconsistent with formulations that predict no effect of size, including the proposal of Harris and Draga (2005) that perceived trajectory is based on total change in visual direction. Instead, we found that the perceptual error can be explained by a simple model in which looming and changing disparity signals are added to give a combined motion-in-depth signal. This is consistent with the theory of Regan and Beverley (1979). The account is sufficient to explain the effect of size on trajectory perception and is attractive from the point of view of computational simplicity compared with alternative higher level cue combination strategies. The scheme predicts perceptual inaccuracies but would be well suited to situations in which rapid visually guided responses must be made, such as interception and collision avoidance.

**Appendix A**

Can distance-scaling errors explain the results?

Several theories of trajectory perception outlined in the Introduction section (Equations 8–10) propose that trajectory perception involves scaling retinal information using distance information. Thus, one possibility may be that the pattern of apparently parallel trajectory misestimates shown in Figures 4 and 5 is due to errors in perceived distance used to scale retinal information (Rushton & Duke, 2007, 2009). Since retinal size can act as a distance cue (e.g., Collett et al., 1991; Sousa et al., 2011), it is possible that perceived distance to the small object was larger than that for the larger one. Informal observation of the stimuli did not suggest this was the case; however, we show below that the predicted direction of the effect is the reverse of the observed effect so we can rule out this possibility as an account of the findings.

The perceived trajectory angle, $\beta^\prime$, is related to trajectory angle $\beta$ as shown in Equation A1, where $D$ and $D'$ are the actual and perceived distances to the trajectory start point, respectively. $A1$ predicts that perceived trajectory angle is underestimated when perceived distance is overestimated:

$$\tan \beta^\prime = \frac{D}{D'} \tan \beta. \quad (A1)$$

When the perceived trajectory angles of the reference and test spheres are equal as in our apparently parallel judgment task, then the physical test trajectory angle is related to the physical reference angle by

$$\tan \beta_{\text{test}} = k \tan \beta_{\text{reference}}, \quad (A2)$$

where

$$k = \frac{D'_{\text{test}}}{D_{\text{reference}}}. \quad (A3)$$

Similarly, physical test and reference crossing distances are related by

$$X_{\text{c test}} = kX_{\text{c reference}}. \quad (A4)$$
So, if the distance to a small test object is overestimated relative to a large reference object, then the physical test trajectory crossing distance must be set to a larger value than the physical reference crossing distance. This prediction is in the opposite direction from our observed data; therefore, we can rule out an interpretation of the results data based on misestimation of distance.

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