Comparison of two weighted integration models for the cueing task: linear and likelihood

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In a task in which the observer must detect a signal at two locations, presenting a precue that predicts the location of a signal leads to improved performance with a valid cue (signal location matches the cue), compared to an invalid cue (signal location does not match the cue). The cue validity effect has often been explained with a limited capacity attentional mechanism improving the perceptual quality at the cued location. Alternatively, the cueing effect can also be explained by unlimited capacity models that assume a weighted combination of noisy responses across the two locations. We compare two weighted integration models, a linear model and a sum of weighted likelihoods model based on a Bayesian observer. While qualitatively these models are similar, quantitatively they predict different cue validity effects as the signal-to-noise ratios (SNR) increase. To test these models, 3 observers performed in a cued discrimination task of Gaussian targets with an 80% valid precue across a broad range of SNR's. Analysis of a limited capacity attentional switching model was also included and rejected. The sum of weighted likelihoods model best described the psychophysical results, suggesting that human observers approximate a weighted combination of likelihoods, and not a weighted linear combination.

Keywords : cueing, Bayesian observer, selective attention

Introduction

The cueing task has been an important paradigm in the study of attention, and has had broad applications in vision, cognitive psychology, and cognitive neuroscience (see Pashler, 1998, or Posner & Peterson, 1990, for a review). In a simple typical cueing task, observers are asked to detect a signal appearing one of two locations. A cue appears prior to the signal stimulus at one of the two locations, and typically predicts the probable appearance of the signal at the cued location. Besides the simple cueing task, there have been many variations of the cueing paradigm that have been studied, such as changing the number of locations and the validity of the precue. The common finding in the cueing task is that performance, measured either as accuracy or response time, is better in those trials in which the cue is valid (signal appears at the cued location) than the trials in which the cue is invalid (signal appears at the uncued location). The cue validity effect, or cueing effect, has often been explained by assuming that attention is a limited-capacity resource, and that attention is drawn or allocated to the precue location. Limited capacity is often implied by descriptions of attention enhancing the stimulus at the cued location (e.g., Eriksen & James, 1986; Posner, 1980; Allport, 1987; Posner & Peterson, 1990; Spitzer, Desimone, & Moran, 1988; Henderson, 1996; Luck, et al., 1994; Luck, Hillyard, Mouloua, & Hawkins, 1996), or degrading the stimulus at the uncued location (e.g., Broadbent, 1958; Kahneman, 1973; Treisman, 1992). Common amongst these descriptions is the concept that attention improves the perceptibility, or quality of processing, of the signal at the cued location, relative to the uncued location. (It should be noted that some of these authors’ descriptions (e.g., Posner, 1980; Broadbent, 1958), while implying limited capacity, might be interpreted more generally to include the weighted integration models discussed later.) Several authors (Shaw, 1980; Kinchla, Chen, & Evert, 1995; Sperling, 1984; Sperling & Dosher, 1986; Shiu & Pashler, 1994, 1995; Eckstein, Shimozaki, & Abbey, 2002; Shimozaki, Eckstein, & Abbey, 2001) have noted that a cueing effect can be predicted by an unlimited capacity model that has the same perceptual quality at the cued and uncued locations. These models represent a challenge to limited capacity attentional models of the cueing paradigm, which imply a change in perceptual quality across the cued and uncued locations. The two key components of the unlimited capacity models are first, attention only is a selective mechanism, and second, the response at each location is perturbed by internal noise. Because of the stochastic nature of the internal response, these models...
are often considered Signal Detection Theory (SDT) models (Green & Swets, 1974). Another well-known result in attention that also has been modeled with this approach is the decrease in performance with increasing number of items in a visual search task, also known as the set-size effect (Kinchla, 1974, 1977; Shaw, 1980, 1984; Eckstein, et al., 2000; Palmer, 1995; Palmer, et al., 1993; Palmer, et al., 2000; Verghese, 2001). As with the cueing effect, the set-size effect has often been attributed to various limited capacity attentional mechanisms, and the SDT models of visual search challenge this attribution.

Two different unlimited capacity models have been proposed based on the concepts developed in SDT. The first is a weighted linear model proposed by Kinchla, et al., (1995). In this model, the response at each location is perturbed by noise, and the weighted responses are linearly combined to form a single decision variable. The second model is a sum of weighted likelihoods model based on a Bayesian optimal observer (Eckstein, et al., 2002; Shimozaki, et al., 2001). In this model, the response at each location are assumed to be noisy, the likelihood of the data given target presence is calculated for each of the two locations (cued and uncued), and a weighted sum of the likelihoods is computed to form a single decision variable. Both models weight information differentially, with the cued location being weighted more heavily. Because of this differential weighting, both models predict a cueing effect with the same sensitivity (d') at the cued and uncued locations. The equivalent sensitivity implies that these models do not change their perceptual quality at the cued (attended) location, and therefore, they can be considered unlimited capacity models. These models are also known as selective attention models (Graham, 1989; Palmer, Ames, & Lindsey, 1993; Palmer, 1995), as attention only determines the weights in these models, or how information is selected, or chosen, for the task.

While the two weighted integration models (weighted linear and Bayesian weighted likelihood) predict cueing effects that are qualitatively similar, it can be shown that these models differ substantially in their quantitative predictions of the size of the cueing effect as a function of signal-to-noise ratio (SNR). In particular, the weighted linear integration model predicts larger cueing effects than the Sum of Weighted Likelihoods Bayesian model as target/non-target discriminability increases. Therefore, from an empirical point of view, it seems important to determine whether human observer performance is consistent with one or the other model.

In addition, in some cases the Sum of Weighted Likelihoods model is equivalent to the optimal Bayesian (ideal) observer, or, in other words, it predicts the best possible performance for the cueing task across both valid and invalid cue trials. Because of this, it can be used as a standard of objective comparison (Shimozaki, et al., 2001). Finally, Bayesian models have been used successfully to model many aspects of human visual perception, starting with simple detection and discrimination (Green & Swets, 1974; Barlow, 1978; Burgess, Wagner, Jennings, & Barlow, 1981; Kersten, 1984; Eckstein, Ahumada, & Watson, 1997), including motion (Weiss & Adelson, 1998), texture (Knill, in press), object recognition (Braje, Tjan, & Legge, 1995; Liu, Knill, & Kersten, 1995; Tjan, Braje, Legge, & Kersten 1995), color constancy (Brainard & Freeman, 1994), perceptual learning (Gold, Bennett, & Sekuler, 1999; Abbey, Eckstein, & Shimozaki, 2001), heading (Crowell & Banks, 1996), and reading (Legge, Kitz, & Tjan, 1997). Therefore, also from a theoretical point of view, it seems important to distinguish whether human performance in the cueing paradigm is consistent with a Bayesian type model or a linear weighted integration model.

To test these models, human observers performed in a cued discrimination task of Gaussian signals across a range of signal-to-noise ratios, and the models were fit to their results. Also, analysis of a third 'attentional switching' limited capacity model is included, which assumes that the observer switches his or her attention to either the cued or the uncued location with a certain probability from trial to trial. If attention is not placed at the target location, the model assumes that the observer is at chance performance.

**Description of Models**

**Weighted Linear Integration**

![Figure 1. The weighted Linear model of the cueing task. A yes/no cued detection task is depicted in which the observer must report signal presence at either location. A valid signal present trial (both cue and signal on the left) is shown. The schematic starts on the left with the responses to the cued (x_c) and uncued (x_u) locations.](https://jov.arvojournals.org/)

The linear model was proposed by Kinchla, first to explain set size effects (Kinchla, 1974, 1977), and then applied to cueing effects (Kinchla, Chen, & Evert, 1995). The schematic in Figure 1 describes the behavior of the model for a cued detection task in which the observer must report signal presence at either of the two locations (and not the location of the signal). First, a response is generated at each location (x_c, x_u), perturbed by internal noise (N). Each response is weighted (scaled) by a separate weighting factor (w_c, w_u), and then summed to give a single decision variable (y), which is compared to a
criteria to make a decision on signal presence. As both \( x_c \) and \( x_{uc} \) are assumed to be Gaussian-distributed, \( y \) is also Gaussian-distributed. The mathematical expressions to obtain performance for the weighted linear integration model are developed in Appendix B.

**Sum of Weighted Likelihoods (Bayesian)**

![Figure 2. The Sum of Weighted Likelihoods model of the cueing task. A valid signal present trial (both cue and signal on the left) of a yes/no cued detection task is depicted. The schematic starts on the left with the responses to the cued (\( x_c \)) and uncued (\( x_{uc} \)) locations.](Image 37x547 to 98x606)

The second model is a Sum of Weighted Likelihoods model based on a Bayesian observer (Green and Swets, 1974), and has been presented previously by Eckstein, et al. (2002), and Shimozaki, et al. (2001). Figure 2 gives a schematic of the model, and Appendix C gives the mathematical equations for predicting performance for the model. As in the linear model, a response is generated at each location (\( x_c, x_{uc} \)), and perturbed by internal noise (N). Then, the model determines the likelihoods of the responses \( x_c \) and \( x_{uc} \), given a signal at the cued location (upper branch, valid trial, signal at cued location = \( s_c \), noise at uncued location = \( n_{uc} \)), given a signal at the uncued location (middle branch, invalid trial, noise at cued location = \( n_c \), signal at uncued location = \( s_{uc} \)), and given signal absence (lower branch, noise at cued location = \( n_c \), noise at uncued location = \( n_{uc} \)). The likelihoods are computed with assumed probability density functions for \( x_c \) and \( x_{uc} \) given signal presence and absence; these probability density functions are assumed to be Gaussian. The likelihoods of the responses given a signal at the cued and uncued locations are then weighted separately (\( w_c, w_{uc} \)) and summed to give an overall weighted likelihood given signal presence (weighted \( L_s \)). \( L_n \) is divided by the likelihood of the responses given signal absence (\( L_n \)), resulting in a weighted likelihood ratio (\( L_s/L_n \)). The weighted likelihood ratio is then compared to a criterion to make a decision on signal presence.

The resulting decision variable includes the sum of the weighted likelihoods (exponentials, because Gaussian probability distributions are assumed), and not the responses themselves. Therefore, unlike the linear model, the decision variable for the Sum of Weighted Likelihoods model is not normal but log-normal. Also, when the weights are equal to the actual prior probabilities of target appearance at the cued and uncued locations (the cue validity), this model is an optimal decision (Bayesian observer) model that maximizes performance over all trials. Thus, in this special case, this model can be used as a comparison standard, relative to human performance (Shimozaki, et al., 2001), a status that cannot be applied to the linear model.

**Attentional Switching**

![Figure 3. The Attentional Switching model of the cueing task. A valid signal present trial (both cue and signal on the left) of a yes/no cued detection task is depicted. The schematic starts on the left with the responses to the cued (\( x_c \)) and uncued (\( x_{uc} \)) locations.](Image 37x547 to 98x606)

The last model is a limited capacity model, and is based on the observer changing attentional strategies from trial to trial by choosing to attend to either the cued or uncued location with a certain probability (e.g., Sperling & Melchner, 1978; Shaw, 1982, the 'all-or-none mixture model'). This model's predictions are qualitatively similar to those of the unlimited capacity models, and is therefore difficult to eliminate from consideration in many cases. For example, a previous study employed the use of classification images in the cueing task, and developed a technique for distinguishing between the general class of selective, unlimited capacity models of attention, and models where attention improves the tuning of the perceptual filter (Eckstein, et al., 2002). The classification image technique, however, could not distinguish between selective attention models and the attentional switching model. However, it can be shown that this model can be distinguished psychophysically in its performance predictions from the
unlimited capacity models over a broad range of signal-to-noise ratios (see next section).

In the attentional switching model (Figure 3, Appendix D), the observer can only attend to one location at a time, and chooses to attend the cued location (in red) with a certain probability (switch). On the other trials (1-switch), the uncued location is attended (in blue). Sensitivity (d’) at the unattended location is assumed to be zero, whereas sensitivity at the cued location is assumed to be nonzero. Thus, this can be considered a limited capacity model, as the sensitivity at the cued and uncued locations differ. At both locations, an internal response is generated (x_{att}, x_{unatt}) that is perturbed by internal noise, and that is assumed to be Gaussian-distributed. Because the sensitivity at the unattended location is zero, response at the unattended location is ignored, and the model only uses the response at the attended location. That value (x_{att}) is then compared to a criterion to make a decision on signal presence.

**Comparisons of Model Predictions as a Function of Signal-to-Noise Ratio**

Figures 4a, 4b, and 4c show the predicted hit and false alarm rates for each of the models in a cueing paradigm with an 80% valid precue. The x-axes are expressed as the Signal-to-Noise Ratio (SNR), which is equivalent to the sensitivity (d’) of the models in a simple discrimination task at a single location. For the weighted integration models, the SNR’s at the cued and uncued locations were the same; thus, these models are unlimited capacity, as there is no change in perceptual quality across the cued and uncued locations. Also, the weights of the weighted integration models were chosen to maximize performance across all conditions. For the Sum of Weighted Likelihoods model, this weight is the cue validity; also, the Sum of Weighted Likelihoods model is the ideal observer in this case. For the Linear model, the weight at the cued location (w_c) that maximized performance was 0.62. For the attentional switching model, the optimal performance is obtained by not switching attention, and maintaining attention on the cued location. As this is a relatively trivial simulation, instead the attentional switching model in this simulation was chosen so that the switching probability matched the cue validity, or 80%.

Figure 5 shows the cueing effects for each of the models, expressed as the difference in the hit rate for the valid trials and the hit rate for the invalid trials. First, it should be noted that both weighted integration models show a cueing effect with the same perceptual quality at the cued and uncued locations. This shows that a cueing effect, by itself, does not suggest a limited capacity attentional mechanism (Shaw, 1980; Kinchla, et al., 1995; Sperling, 1984; Sperling & Dosher, 1986; Shiu & Pashler, 1994, 1995; Eckstein, et al., 2002; Shimozaki, et
Second, the size of cueing effect across signal-to-noise ratios varies differentially for the three models. The attention switching model shows a continuous increase in cueing effect with SNR, with an asymptote surpassing the cueing effects of the weighted integration models. Note that using a switching probability other than the 80%, or simply reducing the sensitivity at the unattended location compared to the attended location instead of setting it to zero, would not change the qualitative aspect of the increasing cueing effect with SNR, only the absolute values of the asymptotes. The two weighted integration models show first an increase, then a decrease in their cueing effects, with the Sum of Weighted Likelihoods model having its peak cueing effect at a smaller SNR. Also, compared to the Sum of Weighted Likelihoods model, the linear model shows a smaller cueing effect at smaller SNR’s, and a larger cueing effect at larger SNR’s. Thus, it may be possible to distinguish between the three models by measuring the cueing effects across a broad range of SNR’s.

**Methods**

To test the three models of the cueing paradigm, three female observers (AH, age 22; KC, age 21; LL, age 21) participated in a cued contrast discrimination task of increment Gaussian disks (σ = 12.4° visual angle) presented for 50 msec in white noise (σ = 4.88 cd/m^2, N.S.D. = 1.62X10^5 deg^2, mean = 25.0 cd/m^2) over a large range of ideal observer signal-to-noise ratios (0.74 to 10.4, computed directly from the image statistics, see Appendix A) manipulated by changing the contrast increment of the Gaussian signal added to a pedestal of 6.25% peak contrast ((peak luminance - mean luminance)/mean luminance).

![Figure 6. Types of trials in the experiment.](https://jov.arvojournals.org/)

The left column of Figure 6 describes a single ‘signal present’ trial in which the cue was ‘valid’. The observer initiated each trial by pressing a key on a computer keyboard. One second after the key press, a square precue (5.86 cd/m^2, side length = 2.5°) appeared for 150 ms around one or both of the potential signal locations (centered 2.5° to the right and left of the center of the display). The validity and the number of the precues determined the condition for that trial, as explained below. Immediately following the precue, the stimulus display appeared for 50 ms. As the signal appeared at cued location in the left column of Figure 1, the figure represents a ‘valid’ signal present trial. Overall, half the trials were signal present trials, with the other half being signal absent trials. The stimulus plus cue duration of 200 ms was chosen to negate the effects of saccades, which typically have a latency of approximately 200 ms. A white noise mask immediately followed for 100 ms, having the same mean background luminance (25.0 cd/m^2) and twice the contrast (σ = 9.76 cd/m^2) as the noise fields in the stimulus displays. The observers then pressed one of two keys on a computer keyboard to indicate their decision for signal presence on that trial. A feedback interval of 400 ms followed, visually indicating if the observer was correct.

The validity of the precue determined the condition of each trial, with the valid trials comprising approximately 80% of the signal present trials, and with the remaining 20% of the trials being ‘invalid’, in which the signal appeared at the uncued location (second column, Figure 6). Thus, the ‘cue validity’ was approximately 80% for this study. There was only one type of signal absent trial, as without the presence of the signal, the ‘valid’ and ‘invalid’ signal absent trials were identical.

A small fixation cross (0.5° by 0.5°, 5.86 cd/m^2) appeared continuously in the center of the display. Also,
to reduce the observer’s intrinsic uncertainty of the signal locations, four small dark lines (5.86 cd/m$^2$, length = 0.5°, width = 0.034°) were continuously displayed near the potential locations of the signal (nearest point 0.5° from center of the signal location).

Separate studies were run at each SNR, comprised of approximately 250 invalid trials, approximately 1000 valid trials, and approximately 1250 signal absent trials for each observer. The trials were broken into 10 sessions of 250 trials each, with the valid, invalid, and signal absent trials randomly intermixed. The types of trials were determined by sampling with replacement; thus the approximate division of trials in each session was 25 invalid, 100 valid, and 125 signal absent trials. For KC and AH, approximately 50 neutral trials (50% valid precues at each location) were also included in each session; these neutral trials were not used in the subsequent analysis.

Within a session, the order of the signal present trials was randomized, as was the placement of the signal between the left and right locations. Hit rates (correctly detecting the signal when present) for the valid and invalid trials and an overall false alarm rate (incorrectly stating signal presence on a signal absent trial) were computed for each session. Standard errors of the mean for the hit and false alarm rates were based on the values over the ten sessions for each SNR.

Stimuli were presented on a monochrome monitor (viewing size = 32.51 by 24.38 cm, resolution = 1024 by 768 pixels, Image Systems Corp., Minnetonka, MN, 55343), sitting 50 cm from the observer. At this distance, each pixel subtended 0.034° of visual angle. Luminance calibrations were performed with software and equipment from Dome Imaging Systems, Inc. (Luminance Calibration System, Waltham, MA).

**Results**

Figure 7 gives the results for the three observers in terms of hit and false alarm rates. As expected, all observers had an improvement in performance with increasing SNR, as shown by the increasing hit rates and decreasing false alarm rates. Figure 8 gives the cueing effects for the human observers, computed as the difference between the hit rates for the valid and invalid trials. All observers first had an increase in the cueing effect, followed by a decrease, as SNR increases. KC had a somewhat smaller cueing effect than AH and LL across all SNR’s.

**Model Fits**

Models fits were found by finding the minimum $\chi^2$ error for the valid and invalid hit rates, and the false alarm rates for each observer separately. For the two weighted integration models, a single parameter defined the weight placed on the cued location relative to the
uncued location ($w_c$). For each observer fit, the weighting parameter was fixed across SNR’s so that only one free weighting parameter was allowed for each observer. A linear relationship was assumed between human sensitivity and image SNR; thus, a slope and intercept were estimated to predict the human sensitivity as a linear function of image SNR. (A fit using a non-linear relationship between $d'$ and image SNR (Eckstein et al., 1997) was not attempted given the relatively good results obtained with the linear function.) For each observer’s results, and for each model, there were 9 free parameters, 1 weight of the information at the cued location ($w_c$), one slope ($b$) and intercept ($a$) relating the human index of detectability ($d'$) and image SNR, and 6 decision criteria ($crit_i$), one for each image SNR, to estimate 18 data values, 6 valid hit rates, 6 invalid hit rates, and 6 false alarm rates. For the attentional switching model, the fits were done with the switch probability (switch) as a free parameter instead of a weighting parameter.

The fits for the three observers are shown in Figure 9, and Table 1 in Appendix E gives the estimated parameters from the fits. The fits for the attentional switching model are not shown, as they were exceedingly poor, with all $\chi^2$ values greater than 1932. As the attentional switching model predicts a continuous increase in the cueing effect with SNR (see Figure 5), a pattern not indicated by any observer (see Figure 8), the poor fits were not surprising. Across all three observers, the Sum of Weighted Likelihoods model gave the better fits, compared to the linear model, particularly for intermediate SNR values. This is due to the fact that the linear model could not reconcile the relatively large cueing effects for the intermediate SNR’s and the relatively small cueing effects for the larger SNR’s found for the human observers. For both models, the estimated weight for KC was smaller than the weights for other two observers, consistent with KC’s smaller cueing effect.

Figure 8. Cueing effects for the human observers, defined as valid hit rate ($H_v$) – invalid hit rate ($H_i$). Error bars are standard errors of the mean.

Figure 9. Fits of the models to hit and false alarm rates for three observers.
Note that all fits for the false alarm rates were relatively good for both models, and therefore, the differences in the fits were nearly all in the valid and invalid hit rates. Therefore, for a better comparison of the fits, Figure 10 gives the fits for the predicted cueing effects as the difference between the valid and invalid hit rates. Here the advantage for the Sum of Weighted Likelihoods model is clearly seen for all observers throughout all SNR values, particularly at the intermediate values. Table 1 in Appendix A indicates that the best fits for the Sum of Weighted Likelihoods model led to predictions of decreasing criteria with increasing for AH and LL. This represents a deviation from the optimal criterion of zero (expressed as log(likelihood ratio), and for the same number of signal present and signal absent trials), and suggests that AH and LL became increasingly conservative in their judgments as SNR increased.

General Discussion

Limited Capacity versus Weighted Integration Models

A limited capacity model was proposed that assumed an attentional switching strategy between the cued and uncued locations from trial to trial. With a certain probability, this model either chooses to follow the cue, or to attend the uncued location, and performance is assumed to be at chance at the unattended location. The attentional switching model predicts a cueing effect that increases asymptotically with SNR, unlike the cueing effects for the human observers, which first increased, then decreased, with SNR. Thus, the attentional switching model was clearly rejected for this study.

The attentional switching model is a specific version of a limited capacity model, and the proposition that an observer switches a unitary attentional mechanism from trial to trial might be seen as somewhat unlikely. Another general limited capacity model might propose that attention weights the information at each location equally, and induces a change in the tuning of the perceptual filters at the cued and uncued location, such that the perceptual filter at the cued location more closely matches the signal. This more general ‘tuning’ model captures the essential qualities of most descriptions of a limited capacity attentional mechanism (Eriksen & James, 1986; Posner, 1980; Allport, 1987; Posner & Peterson, 1990; Spitzer, et al., 1988; Henderson, 1996; Broadbent, 1958; Kahneman, 1973; Treisman, 1992; Luck, et al., 1994; Luck, et al., 1996), and has been tested with the same cueing task with the classification image technique (Eckstein, et al., 2002). This technique allows the investigator to estimate the shape of the perceptual filters or templates from the observer’s trial to trial decisions and the image noise samples.

Figure 10. Fits of the models to cueing effects for three observers. Cueing effect is given as valid hit rate $H_v$ – invalid hit rate $H_i$. 

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In the cueing paradigm, it can be shown that a selective (weighting) attentional mechanism and a limited capacity tuning attentional mechanism lead to differential changes in the classification images. Roughly speaking, a weighting mechanism leads to simple scalar (magnitude) changes in the classification images, with the classification image for the cued location having the greater magnitude. The changes predicted from an attentional tuning mechanism, however, lead to qualitative ‘shape’ changes between the cued and uncued classification images that cannot be simply scaled into each other. The evidence from the four observers in Eckstein, et al. (2002) strongly supported scalar changes in the classification images at the cued and uncued locations, as opposed to changes in shape. Thus, the perceptual tuning version of a limited capacity model of attention can be discounted for this cueing task, based on the study of classification images.

Bayesian Sum of Weighted Likelihoods versus Weighted Linear Integration

The two weighted integration models, linear and likelihood, have theoretically significant qualitative similarities. Both assume a weighted integration of noisy information across the cued and uncued locations, and propose an attentional mechanism that is purely selective, or unlimited capacity, with no change of perceptual quality at the cued and uncued locations. Notably, both predict a cueing effect without a limited capacity attentional mechanism. While this issue has been discussed by several authors (Shaw, 1980; Kinchla, et al., 1995; Sperling, 1984; Sperling & Dosher, 1986; Shiu & Pashler, 1994, 1995; Eckstein, et al., 2002; Shimozaki, et al., 2001), these models are still an important demonstration, given the pervasiveness of limited capacity models of the cueing effect. There are also, however, important differences between the two models. First, the Sum of Weighted Likelihoods model has the theoretical advantage that, in certain circumstances, it is equivalent to an optimal Bayesian rule, and may be used as a standard of comparison (Shimozaki, et al., 2001). The optimal weight for the Sum of Weighted Likelihoods model is the cue validity, and the cueing effects in this case may be taken as a ‘boundary’ condition. Cueing effects less than or equal to the cueing effects found with the optimal weights do not require the proposition of a limited capacity attentional mechanism. This use of the Bayesian observer in the cueing paradigm is similar to the use of SDT ‘uncertainty’ models for set-size effect in visual search (Shaw, 1980, 1984; Eckstein & Whiting, 1996; Eckstein, 1998; Eckstein, et al., 2000; Palmer, 1995; Palmer, et al., 1993; Palmer, et al., 2000; Verghese, 2001). In those cases, the SDT model predicts a set size effect with an unlimited capacity attentional model. Thus, any set size effect equal to or less than the SDT prediction can also be explained without a limited capacity attentional mechanism.

Second, there are relatively large differences in the predicted cueing effects as a function of SNR for the two models (see Figure 5). While both weighted integration models gave improved fits relative to the attentional switching model, the Sum of Weighted Likelihoods model clearly gave the best fits to the observers’ data. Thus, for this simple cued discrimination task, the observers’ performance was best described by the Sum of Weighted Likelihoods model. As seen in Figure 5, the Sum of Weighted Likelihoods model differs largely from the linear model in the size of the cueing effect at larger SNR’s, with the linear model predicting the larger cueing effect, apparently due to underpredicting the invalid hit rate at the larger SNR’s. Thus, it appears that the linear weighting rule penalizes information from the uncued location too heavily, compared to the Sum of Weighted Likelihoods model at high SNR’s.

Overall, the combination of the classification image technique in Eckstein, et al. (2002) and the modeling of psychophysics in the current study can be a powerful method in assessing the perceptual mechanisms involved in the cueing paradigm. For a simple cued discrimination task, the classification image analysis can discriminate between an attentional tuning model and a selective weighting attentional model, and the evidence from Eckstein, et al. (2002) strongly indicates the weighting hypothesis. The modeling of performance in the current study distinguishes among a limited capacity attentional switching model and two weighted integration models, a distinction that might be difficult within the classification image technique. The results from this study suggest that the Sum of Weighted Likelihoods model best describes performance. We believe that employing these methodologies and others (including those done based on varying the image noise and measuring efficiency, see Dosher & Lu, 2000a, 2000b; Lu & Dosher, 1998, 2000; Gold, et al., 1999) in concert may lead to a clearer understanding of the cueing paradigm.

Weighted Integration Versus Maximum-Value Models

Common variants of the integration models are known as ‘maximum-value’ models, in which the maximum value for a particular decision variable is chosen amongst a number of locations or alternatives. These include ‘spatial uncertainty’ models of visual search (Shaw, 1980, 1984; Eckstein, 1998; Eckstein, et al., 2000; Palmer, 1995; Palmer, et al., 1993; Palmer, et al., 2000; Verghese, 2001), and models of summation across channels or features (i.e., probability summation, Graham, 1989; Tyler & Chen, 2000), and in some cases, they are seen as approximations to ideal observer models (Nolte & Jaarsma, 1967; Palmer, et al., 2000). Appendix F compares the predicted cueing effect of the two
weighted integration models to their analogous maximum-value models.

The first section of Appendix F describes a comparison of the Linear model to a corresponding Maximum of Weighted Responses model, summarized in Figure 12. This figure depicts the Linear model with the same weighting as in Figures 4a and 5 (0.62), along with the Maximum of Weighted Responses model with different weightings. First, it is apparent that a greater weighting of the cued location is necessary for the maximum-value model for a comparably sized cueing effect for the Linear model. Second, the maximum-value model's cueing effects rises less steeply with increases in SNR, such that it had a smaller cueing effect for lower SNR's across all weightings. Thus, the maximum-value model does not appear to be a good approximation of the Linear model. Also, part of the difficulty of the Linear model in fitting the human observer's data was that the Linear model's cueing effect did not rise quickly enough with SNR. As the maximum-value model rises even less quickly, this model would provide worse fits to the human data.

The second section of Appendix F describes the comparison of the Sum of Weighted Likelihoods model with an analogous maximum-value model of weighted likelihoods. This comparison is summarized in Figure 14, with Figure 14a showing the hit and false alarm rates, and Figure 14b showing the cueing effects, for the same weighting depicted in Figures 4b and 5 (0.80). The Maximum Likelihood model approximated the Sum of Weighted Likelihoods model well for high SNR's, and less well for intermediate SNR's. This result agrees with a previous study by Nolte and Jaarma (1967), suggesting that a maximum-value decision rule closely approximates the optimal Bayesian decision rule at higher SNR's for an m-AFC task.5

### Relationship to Previous ‘SDT’ (Uncertainty) Models of Visual Search

The weighted integration models for the cueing paradigm are closely related to the SDT models for set-size effects in visual search, and, in fact, could be seen as an extension of these models (e.g., Kinchla, 1974, 1977; Shaw, 1980, 1984; Eckstein, et al., 2000; Palmer, 1995; Palmer, et al., 1993; Palmer, et al., 2000; Verghese, 2001). (Also, an SDT model predicting the feature/conjunction dichotomy in visual search can be found in Eckstein (1998) and Eckstein, et al. (2000)). Both classes of models for visual search and the cueing tasks are unlimited capacity selective attention models operating on responses perturbed by noise. Also, as noted in the Introduction, the SDT models of visual search and Sum of Weighted Likelihoods model can be employed similarly in setting an upper bound for the expected set size and cueing effects for an unlimited capacity attentional model. However, the weighted integration models are not equivalent to the SDT models for visual search, as the weighted integration models explain cueing effects, and not set-size effects. These two effects, besides being two of the more important phenomena in attention, have also been treated as distinct phenomena, and therefore we believe that they deserve separate and distinct treatments. The models for set-size effects, in essence, assume an equal weighting across locations, and would need to expand to include unequal weightings to model cueing effects (as has been done by Kinchla in his linear model, for both set-size (Kinchla, 1974, 1977) and cueing effects (Kinchla, et al., 1995)). Another difference is that the previous visual search models in general employ a maximum-value decision rule (e.g., Shaw, 1980, 1984; Eckstein, 1998; Eckstein, et al., 2000; Palmer, 1995; Palmer, et al., 1993; Palmer, et al., 2000; Verghese, 2001), choosing the single location most likely to contain the signal, as opposed to integrating information across locations. As mentioned above, in general these maximum-value models of visual search are an approximation to the optimal Bayesian observer, and they become indistinguishable at high SNR’s (Nolte & Jaarsma, 1967; Appendix F).

In a study of a visual search task of orientation discrimination, one study (Baldassi & Verghese, 2002) tested two models analogous to the weighted integration models in the current study, a linear combination rule and a maximum-output rule that approximates a Bayesian observer. In their task, observers were asked to identify an oriented Gabor patch, tilted either clockwise or counterclockwise relative to vertical distractor Gabors. Analogous to the current study, both combination rules predicted a set-size effect without a limited capacity attentional mechanism; however, the two models gave different predictions for the psychometric functions for increasing set size. Notably, the linear combination gave poorer fits to the human observer psychometric functions than the maximum-output approximation of a Bayesian observer. Smith (1998) also tested unweighted linear integration and maximum-value decision rules based on visual search uncertainty models in a cueing paradigm, and a difference of fits between the two types of models could not be distinguished. The unweighted combination rules led to the conclusion that the cueing effects found in the study were due to differences in discriminability at the cued and uncued locations.

### Conclusions

Three models have been presented for the cue validity effect in the cueing paradigm, two weighted integration models, linear and Bayesian likelihood, and a limited capacity attentional switching model. The first two models are selective, unlimited capacity models that may be categorized loosely as SDT models, and have two key components. First, information is integrated across all
locations, and second, the information at the cued location is weighted more heavily. Both models predict a cueing effect while having the same perceptual quality at the cued and uncued locations, contradicting the notion that a cueing effect implies a limited capacity attentional mechanism. While qualitatively similar, analyses of the predicted cueing effect found that the three models gave different predictions with respect to signal-to-noise ratio; to test these predictions, three observers performed in a cued discrimination task across a range of SNR's. For this task, fits for the attentional switching models were poor, and human observer performance was better predicted by a Bayesian model that combines weighted likelihoods than a weighted linear integration of the internal responses at the cued and uncued locations.

\[ w_c = \text{the weight for the cued location} \]
\[ w_{uc} = 1 - w_c = \text{the weight for the uncued location} \]

The model computes a decision variable \( y \) that is the weighted sum of the responses at the cued and uncued locations \( (x_c, x_{uc}) \),

\[ y = w_c x_c + w_{uc} x_{uc} . \]  \hspace{1cm} (B1)

The model assumes that the prior probability of response at each location \( (x_c, x_{uc}) \) is determined by two Gaussian distributions, one for the signal and one for the noise. The mean of the signal distribution is 1, the mean of the noise distribution is zero, and the variance of both distributions is \( \alpha \). These distributions do not have unit variance, unlike the standard SDT assumption, and sensitivity is not described as the distance between the two distributions (\( d' \)); instead, sensitivity in this model is described in terms of the variance. For development of an equivalent model with unit variance, see Eckstein, et al. (2000). Note that \( \alpha \) is the same at the cued and uncued locations, so that this model may be defined as an unlimited capacity model.

### Appendix A. Image Signal-to-Noise Ratio (SNR)

For the case of a signal embedded in white Gaussian noise, the image signal to noise ratio (SNR), which corresponds to the ideal observer index of detectability, can be calculated as follows:

Let \( s(x,y) \) be the array describing luminance profile of the stimulus and \( \sigma_{pixel} \) be the standard deviation of the Gaussian pixel noise. Then the signal energy \( E \) is:

\[ E = \sum_x \sum_y s^2(x,y) . \]  \hspace{1cm} (A1)

In Gaussian (white) noise, image SNR is given by

\[ \text{SNR} = \frac{\sqrt{E}}{\sigma_{pixel}} . \]  \hspace{1cm} (A2)

### Appendix B. Linear Model

This model formulation was derived by Kinchla, et al. (1995), and the terminology and development follow from that study.

Let

\[ x_c = \text{response at cued location} \]
\[ x_{uc} = \text{response at uncued location} \]
\[ s_c = \text{the event of a signal at the cued location} \]
\[ s_{uc} = \text{the event of a signal at the uncued location} \]
\[ n_c = \text{the event of a noise stimulus at the cued location} \]
\[ n_{uc} = \text{the event of a noise stimulus at the uncued location} \]

The expected values of \( y \) for valid trials \( (s_c, n_{uc}) \), invalid trials \( (n_c, s_{uc}) \), and signal absent trials \( (n_c, n_{uc}) \) are as follows:

\[ w_c = \text{the weight for the cued location} \]
\[ w_{uc} = 1 - w_c = \text{the weight for the uncued location} \]

The model computes a decision variable \( y \) that is the weighted sum of the responses at the cued and uncued locations \( (x_c, x_{uc}) \),

\[ y = w_c x_c + w_{uc} x_{uc} . \]  \hspace{1cm} (B1)

The model assumes that the prior probability of response at each location \( (x_c, x_{uc}) \) is determined by two Gaussian distributions, one for the signal and one for the noise. The mean of the signal distribution is 1, the mean of the noise distribution is zero, and the variance of both distributions is \( \alpha \). These distributions do not have unit variance, unlike the standard SDT assumption, and sensitivity is not described as the distance between the two distributions (\( d' \)); instead, sensitivity in this model is described in terms of the variance. For development of an equivalent model with unit variance, see Eckstein, et al. (2000). Note that \( \alpha \) is the same at the cued and uncued locations, so that this model may be defined as an unlimited capacity model.

For a signal stimulus,

\[ p(x|s) = \frac{1}{\sqrt{2\pi \alpha}} \exp \left( -\frac{x^2}{2\alpha} \right) . \]  \hspace{1cm} (B4)

For a noise stimulus,

\[ p(x|n) = \frac{1}{\sqrt{2\pi \alpha}} \exp \left( -\frac{(x-1)^2}{2\alpha} \right) . \]  \hspace{1cm} (B5)

The model predicts performance by evaluating the decision variable \( y \). As \( x_c \) and \( x_{uc} \) are Gaussian distributed, \( y \) is also Gaussian distributed.

The expected values of \( y \) for valid trials \( (s_c, n_{uc}) \), invalid trials \( (n_c, s_{uc}) \), and signal absent trials \( (n_c, n_{uc}) \) are as follows:
\[
\hat{y}_{xc,n_{uc}} = w_{c}\hat{x}_{xc} + w_{uc}\hat{x}_{n_{uc}}
\]
\[
= w_{c}(\mu_{c}) + w_{uc}(\mu_{n_{uc}}) = w_{c}(1) + w_{uc}(0) = w_{c},
\]
(B6)

\[
\hat{y}_{n_{ec}} = w_{c}\hat{x}_{n_{ec}} + w_{uc}\hat{x}_{n_{uc}}
\]
\[
= w_{c}(\mu_{n_{ec}}) + w_{uc}(\mu_{n_{uc}}) = w_{c}(0) + w_{uc}(1) = w_{uc},
\]
(B7)

\[
\hat{y}_{n_{ec},n_{uc}} = w_{c}\hat{x}_{n_{ec}} + w_{uc}\hat{x}_{n_{uc}}
\]
\[
= w_{c}(\mu_{n_{ec}}) + w_{uc}(\mu_{n_{uc}}) = w_{c}(0) + w_{uc}(0) = 0.
\]
(B8)

The variance of \( y \) is given by the following:
\[
\sigma_{y}^{2} = w_{c}^{2}\alpha + w_{uc}^{2}\alpha
\]
(B9)

Thus, a \( d' \) measure can be derived for the valid and invalid trials (compared to the signal absent trials),
\[
d'_{\text{valid}} = \frac{\hat{y}_{xc,n_{uc}} - \hat{y}_{n_{ec},n_{uc}}}{\sigma_{y}}
\]
\[
= \frac{w_{c} - 0}{\left( w_{c}^{2}\alpha + w_{uc}^{2}\alpha \right)^{1/2}} = \frac{w_{c}}{\left( w_{c}^{2}\alpha + w_{uc}^{2}\alpha \right)^{1/2}},
\]
(B10)

\[
d'_{\text{invalid}} = \frac{\hat{y}_{n_{ec},n_{uc}} - \hat{y}_{n_{ec},n_{uc}}}{\sigma_{y}}
\]
\[
= \frac{w_{uc} - 0}{\left( w_{c}^{2}\alpha + w_{uc}^{2}\alpha \right)^{1/2}} = \frac{w_{uc}}{\left( w_{c}^{2}\alpha + w_{uc}^{2}\alpha \right)^{1/2}},
\]
(B11)

To generate the hit and false alarm rates, values of \( y \) were normalized to unit variance, which has no effect on the predictions of the model.
\[
y_{z} = \frac{y}{\sigma_{y}} = \frac{y}{\left( w_{c}^{2}\alpha + w_{uc}^{2}\alpha \right)^{1/2}}.
\]
(B12)

Then, the hit and false alarm rates predicted by the model were found by choosing a criterion (\( \text{crit} \)) so that the normalized \( y \) values above the criterion (\( x_{\text{crit}} \)) led to ‘signal present’ decisions for the model.
\[
H_{v} = \text{valid hit rate} = \Pr(y_{z} > \text{crit} | s_{c}, n_{uc})
\]
(B13)

\[
H_{i} = \text{invalid hit rate} = \Pr(y_{z} > \text{crit} | n_{c}, s_{uc})
\]
(B14)

\[
FA = \text{false alarm rate} = \Pr(y_{z} > \text{crit} | n_{c}, n_{uc})
\]
(B15)

Defining \( g(x) \) as the Gaussian probability density function of unit variance and mean equal to zero, and \( G(x) \) as the cumulative probability density function for \( g(x) \), we can substitute these functions into the above equations.
\[
g(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} x^{2}\right),
\]
\[
G(x) = \int_{-\infty}^{x} g(t) \, dt.
\]

Therefore,
\[
H_{v} = \Pr(y_{z} > \text{crit} | s_{c}, n_{uc}) = 1 - G(\text{crit} - d'_{\text{valid}})
\]
(B16)

\[
H_{i} = \Pr(y_{z} > \text{crit} | n_{c}, s_{uc}) = 1 - G(\text{crit} - d'_{\text{invalid}})
\]
(B17)

\[
FA = \Pr(y_{z} > \text{crit} | n_{c}, n_{uc}) = 1 - G(\text{crit})
\]
(B18)

Appendix C. Sum of Weighted Likelihoods Model

This model is a modification of an optimal Bayesian observer (Green & Swets, 1974), and begins with the human internal response. A version of this model beginning with the image (correlation of perceptual filters with the image) can also be found in Eckstein, et al. (2002).

The model calculates three likelihoods of the responses \( x_{c} \) and \( x_{uc} \).

The likelihood of \( x_{c} \) and \( x_{uc} \) given that a signal stimulus was present at the cued location and a noise stimulus was present at the uncued location (valid trial),
\[
L_{x_{c}, n_{uc}} = p(x_{c} | x_{uc} | s_{c}, n_{uc}) .
\]
(C1)

The likelihood of \( x_{c} \) and \( x_{uc} \) given that a noise stimulus was present at the cued location and a signal stimulus was present at the uncued location (invalid trial),
\[
L_{n_{c}, n_{uc}} = p(x_{c} | x_{uc} | n_{c}, s_{uc}) .
\]
(C2)

The likelihood of \( x_{c} \) and \( x_{uc} \) given that a noise stimulus was present at the cued location and at the uncued location, (signal absent trial),
\[
L_{n_{c}, n_{uc}} = p(x_{c} | x_{uc} | n_{c}, n_{uc}) .
\]
(C3)

Each likelihood is the probability of \( x_{c} \) given the stimulus at the cued location, and the probability of \( x_{uc} \) given the stimulus at the uncued location,
\[
L_{x_{c}, n_{uc}} = p(x_{c} | x_{uc} | s_{c}, n_{uc}) = p(x_{c} | s_{c}) p(x_{uc} | n_{uc}) ,
\]
(C4)

\[
L_{n_{c}, n_{uc}} = p(x_{c} | x_{uc} | n_{c}, s_{uc}) = p(x_{c} | n_{c}) p(x_{uc} | s_{uc}) ,
\]
(C5)

\[
L_{n_{c}, n_{uc}} = p(x_{c} | x_{uc} | n_{c}, n_{uc}) = p(x_{c} | n_{c}) p(x_{uc} | n_{uc}) .
\]
(C6)

The model calculates the weighted likelihoods of the responses given a signal trial (weighted \( L_{v} \) by summing
the weighted likelihoods given a valid signal trial \((s_c, n_{uc})\), and given an invalid signal trial \((n_c, s_{uc})\).

\[
\text{weighted } L_x = w_c L_{c|n_{uc}} + w_{uc} L_{n|s_{uc}}
\]

\[
= w_c p(x_c | s_c)p(x_{uc} | n_{uc}) + w_{uc} p(x_c | n_c)p(x_{uc} | s_{uc}) .
\]  \(C7\)

The model calculates the weighted likelihood ratio of the responses.

\[
L_{s/n} = \frac{\text{weighted } L_x}{L_n}
\]

\[
= \frac{w_c p(x_c | s_c)p(x_{uc} | n_{uc}) + w_{uc} p(x_c | n_c)p(x_{uc} | s_{uc})}{p(x_c | n_c)p(x_{uc} | n_{uc})} .
\]  \(C8\)

The model assumes that the probability density functions of the response at each location are determined by two Gaussian distributions of unit variance, one for the signal and one for the noise. The mean of the noise distribution is zero, and sensitivity is defined as the mean of the signal distribution \((d')\). This is the standard Signal Detection Theory assumption. Note that \(d'\) is the same regardless of whether the location is cued or uncued,

\[
\mu_n = 0 ,
\]

\[
\mu_s = d' .
\]

The index of detectability was related to the ideal observer (image) SNR by the following linear relationship:

\[
d' = b(SNR) + a .
\]  \(C9\)

where \(b\) is a term that includes the suboptimal nature of the human perceptual filter (sampling efficiency, Burgess et al., 1981) and internal noise (Burgess & Colborne, 1988).

Then, for a noise stimulus,

\[
p(x|n) = g(x) .
\]

For a signal stimulus,

\[
p(x|s) = g(x-\mu_s) = g(x-d') .
\]  \(C11\)

Substituting the Gaussian assumption into the weighted likelihood ratio,

\[
L_{s/n} = \frac{w_c p(x_c | s_c)p(x_{uc} | n_{uc}) + w_{uc} p(x_c | n_c)p(x_{uc} | s_{uc})}{p(x_c | n_c)p(x_{uc} | n_{uc})}
\]

\[
= \frac{w_c g(x_c - d')g(x_{uc}) + w_{uc} g(x_c)g(x_{uc} - d')}{g(x_c)g(x_{uc})}
\]

\[
= \frac{-\frac{1}{2}(x_c - d')^2 + x_{uc}^2}{e^{-\frac{1}{2}(x_c^2 + x_{uc}^2)}
\]

\[
\times\left[\frac{-\frac{1}{2}(x_c - d')^2 + x_{uc}^2}{e^{-\frac{1}{2}(x_c^2 + x_{uc}^2)}}\right].
\]  \(C12\)

To find the predicted hit and false alarm rates, the log likelihood ratio was compared to a criterion \((\text{crit})\). (The monotonic log transform of the likelihood ratio has no effect on the predictions of the model.) Responses generating log likelihood ratios greater than the criterion led to 'signal present' decisions for the model. Therefore,

\[
H_v = \text{valid hit rate} = \Pr(\log(L_{s/n})>\text{crit}|s_c,n_{uc})
\]  \(C13\)

\[
H_i = \text{invalid hit rate} = \Pr(\log(L_{s/n})>\text{crit}|n_c,s_{uc})
\]  \(C14\)

\[
FA = \text{false alarm rate} = \Pr(\log(L_{s/n})>\text{crit}|n_c,n_{uc})
\]  \(C15\)

Monte Carlo simulations of 10,000 trials for each SNR were performed to generate predictions for the model. When the weights match the cue validity, the Sum of Weighted Likelihoods model becomes the Bayesian model. If, in addition, \(d'\) is replaced by the image SNR (which assumes that the perceptual filter perfectly matches the signal and there is no internal noise), then the model becomes the optimal observer.

### Appendix D. Attentional Switching Model

This section describes an attentional switching model (Sperling & Melchner, 1978; Shaw, 1982) in which the model chooses to attend to either the cued or the uncued location with a certain probability on each trial.

Let

\[
x_{att} = \text{response at cued location}
\]

\[
x_{unatt} = \text{response at uncued location}
\]

\[
\text{switch} = \text{the probability of attending the cued location}
\]

\[
1 - \text{switch} = \text{the probability of attending the uncued location}
\]

On each trial, this model chooses to attend to either the cued location with a probability equal to \(\text{switch}\) \((p(\text{attend cued}) = \text{switch})\), or the uncued location with a probability equal to \(1 - \text{switch}\) \((p(\text{attend uncued}) = 1 - \text{switch})\). The response at the unattended location \((x_{unatt})\) is assumed to have zero sensitivity, and is ignored. Thus, the decisions of the attentional switching model are made solely on the response from the attended location \((x_{att})\).

As in the Sum of Weighted Likelihoods model, assume that the probability density functions for \(x_{att}\) are described by two Gaussian distributions of unit variance, one for the signal \((\mu_{att} = d')\) and one for the noise \((\mu_{att} = 0)\).

Then, for a noise stimulus,
\[ p(x_{\text{att}} | n) = g(x_{\text{att}}) = \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{x_{\text{att}}^2}{2} \right). \] (D1)

For a signal stimulus,
\[ p(x_{\text{att}} | s) = g(x_{\text{att}} | H_{\text{att}s}) \]
\[ = g(x_{\text{att}} - d^*) = \frac{1}{\sqrt{2\pi}} \exp \left( \frac{(x_{\text{att}} - d^*)^2}{2} \right). \] (D2)

To calculate the hit and false alarm rates, a criterion \((\text{crit})\) was chosen so that attended responses \((x_{\text{att}})\) above the criterion led to ‘signal present’ decisions for the model.

\[ H_s = \text{valid hit rate} = \Pr(x_{\text{att}} > \text{crit} | s, n_{\text{att}}) \] (D3)

\[ H_i = \text{invalid hit rate} = \Pr(x_{\text{att}} > \text{crit} | n, s_{\text{uc}}) \] (D4)

\[ FA = \text{false alarm rate} = \Pr(x_{\text{att}} > \text{crit} | n, s_{\text{uc}}) \] (D5)

These probabilities are comprised of:

A) the probability of \(x_{\text{att}}\) exceeding the criterion when attending the cued location, \(p(\text{attend cued}) = \text{switch}\), and

B) the probability of \(x_{\text{att}}\) exceeding the criterion when attending the uncued location, \(p(\text{attend uncued}) = 1 - \text{switch}\).

Therefore,
\[ H_s = \Pr(x_{\text{att}} > \text{crit} | s, n_{\text{att}}) = \text{switch} \Pr(x_{\text{att}} > \text{crit} | s_c) + (1 - \text{switch}) \Pr(x_{\text{att}} > \text{crit} | n_{\text{uc}}) \] (D6)

\[ H_i = \Pr(x_{\text{att}} > \text{crit} | n, s_{\text{uc}}) = \text{switch} \Pr(x_{\text{att}} > \text{crit} | n_c) + (1 - \text{switch}) \Pr(x_{\text{att}} > \text{crit} | s_{\text{uc}}) \] (D7)

\[ FA = \Pr(x_{\text{att}} > \text{crit} | n, s_{\text{uc}}) = \text{switch} \Pr(x_{\text{att}} > \text{crit} | n_c) + (1 - \text{switch}) \Pr(x_{\text{att}} > \text{crit} | n_{\text{uc}}) \] (D8)

Substituting the cumulative Gaussian probability density function,
\[ \Pr(x_{\text{att}} > \text{crit} | s) = 1 - G(\text{crit} - d^*) , \]
\[ \Pr(x_{\text{att}} > \text{crit} | n) = 1 - G(\text{crit}) . \]

Therefore,
\[ H_s = \text{switch} (1 - G(\text{crit} - d^*)) + (1 - \text{switch})(1 - G(\text{crit})) \] (D9)

\[ H_i = \text{switch} (1 - G(\text{crit})) + (1 - \text{switch})(1 - G(\text{crit} - d^*)) \] (D10)

\[ FA = 1 - G(\text{crit}) \] (D11)

### Appendix E. Model Fit Parameters

#### Table E1. Parameters for the Model Fits.

<table>
<thead>
<tr>
<th>Sum of Weighted Likelihoods</th>
<th>Linear</th>
</tr>
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Human \(d^*\)’s were fit to the linear function \(d^* = b(\text{Image SNR}) + a\). The criteria for the Sum of Weighted Likelihoods model are expressed as the log weighted likelihood ratio \(\log(L_{\text{wLin}})\). The criteria for the Linear model are expressed as the normalized distance from the mean of the hypothesized noise distribution for the weighted linear decision variable \((y)\).

### Appendix F. Comparison of Weighted Integration Models to Maximum-Value Models

A common alternative to a combination rule across locations or alternatives is a rule choosing the single location or alternative with the greatest value for a particular decision variable. Such ‘maximum-value’ rules may be found in the context of, for example, models of set-size effects in visual search (Palmer, et al., 1993; Palmer, 1995), and summation across channels or features (i.e., probability summation, Graham, 1989; Tyler & Chen, 2000). Often these models are viewed as approximations of the weighted integration models, particularly in the case of the ideal observer (Sum of...
Therefore, the valid hit rate \( H_v \) ('signal present' response with the signal at the cued location and the 'noise' (signal absent) stimulus in the uncued location) is given by the following,

\[
H_v = \Pr \left( w_c x_c > w_{uc} x_{nc} \text{ and } w_c x_c > \text{crit} \right) \\
\quad \quad \text{or} \quad \Pr \left( w_{uc} x_{nc} > w_c x_c \text{ and } w_{uc} x_{nc} > \text{crit} \right)
\]

\[
= \int_{\text{crit}}^{\infty} \Pr \left( w_c x_c > w_{uc} x_{nc} \right) p(w_c x_c) dx_c \\
+ \int_{\text{crit}}^{\infty} \Pr \left( w_{uc} x_{nc} > w_c x_c \right) p(w_{uc} x_{nc}) dx_{nc}
\]

\[\text{(F1)}\]

Equivalently, for the invalid hit rate \( H_i \) and the false alarm rate \( FA \), there are the following equations,

\[
H_i = \Pr \left( w_c x_c > w_{uc} x_{nc} \text{ and } w_c x_c > \text{crit} \right) \\
\quad \quad \text{or} \quad \Pr \left( w_{uc} x_{nc} > w_c x_c \text{ and } w_{uc} x_{nc} > \text{crit} \right)
\]

\[
= \int_{\text{crit}}^{\infty} \Pr \left( w_c x_c > w_{uc} x_{nc} \right) p(w_c x_c) dx_c \\
+ \int_{\text{crit}}^{\infty} \Pr \left( w_{uc} x_{nc} > w_c x_c \right) p(w_{uc} x_{nc}) dx_{nc}
\]

\[\text{(F2)}\]

\[\text{FA} = \Pr \left( w_c x_c > w_{uc} x_{nc} \text{ and } w_c x_c > \text{crit} \right) \\
\quad \quad \text{or} \quad \Pr \left( w_{uc} x_{nc} > w_c x_c \text{ and } w_{uc} x_{nc} > \text{crit} \right)
\]

\[
= \int_{\text{crit}}^{\infty} \Pr \left( w_c x_c > w_{uc} x_{nc} \right) p(w_c x_c) dx_c \\
+ \int_{\text{crit}}^{\infty} \Pr \left( w_{uc} x_{nc} > w_c x_c \right) p(w_{uc} x_{nc}) dx_{nc}
\]

\[\text{(F3)}\]

The means and standard deviations for the weighted responses to a signal (s) or noise (signal absent, n) stimulus at the cued and uncued locations are given below. For the cued location,

\[
w_c \hat{x}_c = w_c \hat{x}_n = w_c
\]

\[
\sigma_{w_c x_c} = \frac{w_c \sigma_x}{d'}
\]

\[
w_c \hat{x}_n = 0
\]

\[
\sigma_{w_c x_n} = \frac{w_c \sigma_x}{d'}
\]
For the uncued location,
\[
\begin{align*}
\hat{w}_{uc} \cdot \hat{x}_{uc} &= w_{uc} \cdot \hat{x} = w_{uc} \\
\sigma_{w_{uc} \cdot \hat{x}_{uc}} &= w_{uc} \cdot \sigma = \frac{w_{uc}}{d'}
\end{align*}
\]
\[
\begin{align*}
\hat{w}_{uc} \cdot \hat{x}_{uc} &= w_{uc} \cdot \hat{x} = 0 \\
\sigma_{w_{uc} \cdot \hat{x}_{uc}} &= w_{uc} \cdot \sigma = \frac{w_{uc}}{d'}
\end{align*}
\]

Let \(g(x, \sigma)\) and \(G(x, \sigma)\) be defined as the Gaussian probability density and the cumulative Gaussian probability function, respectively, with a mean of zero and standard deviation of \(\sigma\),
\[
g(x, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right),
\]
\[
G(x, \sigma) = \int_{-\infty}^{x} g(x, \sigma) dx.
\]

Substituting the Gaussian assumptions and the above means and standard deviations, \(H_v\), \(H_i\), and \(FA\) are described as follows,
\[
H_v = \int_{-\infty}^{\infty} G\left(w_{uc} x, \frac{w_{uc}}{d'}\right) g\left(w_{uc} x - \frac{w_{c}}{d'}\right) dx
\]
\[+ \int_{-\infty}^{\infty} G\left(w_{uc} x - \frac{w_{c}}{d'}\right) g\left(w_{uc} x - \frac{w_{uc}}{d'}\right) dx,
\]
\[
H_i = \int_{-\infty}^{\infty} G\left(w_{uc} x - \frac{w_{uc}}{d'}\right) g\left(w_{uc} x - \frac{w_{c}}{d'}\right) dx
\]
\[+ \int_{-\infty}^{\infty} G\left(w_{uc} x - \frac{w_{c}}{d'}\right) g\left(w_{uc} x - \frac{w_{uc}}{d'}\right) dx,
\]
\[
FA = \int_{-\infty}^{\infty} G\left(w_{uc} x, \frac{w_{uc}}{d'}\right) g\left(w_{uc} x - \frac{w_{c}}{d'}\right) dx
\]
\[+ \int_{-\infty}^{\infty} G\left(w_{uc} x - \frac{w_{c}}{d'}\right) g\left(w_{uc} x - \frac{w_{uc}}{d'}\right) dx.
\]

Figure 12 compares the predicted cueing effects of the maximum-value model across a range of weights against the linear model with a weight of 0.62, the same as the weight depicted in Figures 4a and 5 for the Linear model. The criteria were adjusted to give the best performance for the models. First, it can be seen that a larger weighting must be given to the Maximum of Weighted Responses model to approximate the cueing effects for the Linear model. Second, increasing the weight shifts the peak cueing effect to higher SNR’s. Lastly, the functions for the Maximum of Weighted Responses model rises less steeply than the Linear model. The Linear Model provided a relatively poor fit to the observers’ data, compared to the Sum of Weighted Likelihoods model, in part because it rises less steeply than the Sum of Weighted Likelihoods model. Consequently, fits of the observers’ data with the Maximum of Weighted Responses model should be worse than both the Linear Model and the Sum of Weighted Likelihoods model. Overall, the maximum-value model does not appear to be a good approximation to the Linear model.

**Figure 12.** Cueing effects of the Maximum of Weighted Responses model compared to the Linear model. The weights used for the maximum-value model are indicated on the right (0.62 – 0.95); the weight for the Linear model (0.62) is the same as that used in Figures 4a and 5.

**Maximum Likelihood vs. Sum of Weighted Likelihoods**

Figure 13 depicts the Maximum Likelihood model, a counterpart to the Sum of Weighted Likelihoods model. This model finds likelihoods of the signal at the cued location, the signal at the uncued location, and signal absence given the stimulus, and calculates the likelihood ratios of target presence at both locations. The maximum of the weighted likelihood ratios is then chosen and compared to a criterion.
The first step of this model is equivalent to the Sum of Weighted Likelihoods model, with the computation of the likelihoods of target presence at the cued location, target presence at the uncued location, and target absence. If $x_c$, $x_{uc}$, $w_c$, $w_{uc}$, $S_c$, $S_{uc}$, $n_c$, $n_{uc}$, $g(x)$ and $d'$ are defined as before in Appendix C, Sum of Weighted Likelihoods Model, the likelihoods are defined in Equations C4, C5, and C6.

The model then computes the likelihood ratio of target presence at the cued location over target absence, given by the following equations, respectively. (With the appropriate changes to the criteria, using the likelihoods is equivalent to using the likelihood ratios in this case.)

$$L_{n_c,n_{uc}/n_c,n_{uc}} = \frac{w_c p(x_c|s_c)p(x_{uc}|n_{uc})}{p(x_c|n_c)p(x_{uc}|n_{uc})} = \frac{w_c g(x_c-d')}{g(x_c)}$$

$$= \frac{\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x_c-d')^2}{2}\right)}{\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x_c^2}{2}\right)} = \frac{\exp\left(-\frac{(x_c-d')^2}{2}\right)}{\exp\left(-\frac{x_c^2}{2}\right)}$$

(F7)

$$L_{n_c,n_{uc}/n_c,n_{uc}} = \frac{w_{uc} p(x_{uc}|n_c)p(x_{uc}|n_{uc})}{p(x_{uc}|n_c)p(x_{uc}|n_{uc})} = \frac{w_{uc} g(x_{uc}-d')}{g(x_{uc})}$$

$$= \frac{\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x_{uc}-d')^2}{2}\right)}{\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x_{uc}^2}{2}\right)} = \frac{\exp\left(-\frac{(x_{uc}-d')^2}{2}\right)}{\exp\left(-\frac{x_{uc}^2}{2}\right)}$$

(F8)

The maximum of these two likelihood ratios is then compared to a criterion for a decision upon target presence.

**Figure 14** compares the results of Monte Carlo simulations for the Sum of Weighted Likelihoods and the maximum likelihood models, with criteria optimized for the best performance for each model. The weight for each model was 0.80, the optimal (Bayesian) weighting for the Sum of Weighted Likelihoods model, and the same as that used in Figures 4b and 5. Figure 14a shows the hit and false alarm rates, and Figure 14b shows the cueing effects. The maximum likelihood model predicts a larger cueing effect up to a signal-to-noise ratio of about 2.6, with a peak difference of approximately 0.08 at an SNR of about 0.8. As seen in Figure 14a, this is due to the maximum likelihood model predicting a smaller invalid hit rate than the Sum of Weighted Likelihoods model. Thus, the maximum likelihood model tends to underweight information (suboptimally) at the uncued location, due to its choosing of information at one location with the maximum likelihood in its decision. As the signal-to-noise ratio increases, the maximum likelihood model more closely approximates the Sum of Weighted Likelihoods model. This is analogous to results by Nolte and Jaarma (1967) that a maximum-value decision rule closely approximates the optimal Bayesian decision rule at higher SNR’s for an m-AFC task.  

**Figure 15** depicts the difference in the predicted cueing effects across SNR for the two models. The results for a weight of 0.80 in Figure 14 are represented, as well as two other weights. As shown by this figure, the peak difference between the two models tends to shift to higher SNR’s for increasing weights. To conclude, the maximum likelihood model may approximate the Sum of Weighted Likelihoods model for higher SNR’s, and depending upon the specific weight.
Figure 14b. Cueing effects of the Maximum Likelihood model compared to the Sum of Weighted Likelihoods model. The cueing effect is defined as valid hit rate ($H_v$) – invalid hit rate ($H_i$). The weight ($w_c$) for each model was 0.80, the same weight as that used for the Sum of Weighted Likelihoods model in Figures 4b and 5.

Figure 15. Difference in the cueing effects predicted by the Maximum Likelihood and Sum of Weighted Likelihoods models (Maximum Likelihood –Sum of Weighted Likelihoods) across a range of weights. The cueing effect is defined as valid hit rate ($H_v$) – invalid hit rate ($H_i$).

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Footnotes

1 These models have also been called ‘noise reduction’ models (e.g., Shiu & Pashler, 1994, 1995; Henderson, 1996). Note that in this context, ‘noise reduction’ implies an unlimited capacity attentional mechanism, and is not equivalent to similar terms found in limited capacity attentional models. For example, Dosher & Lu (2000a, 2000b; Lu & Dosher, 1998, 2000), in their Perceptual Template Model based on varying image noise to measure efficiency (Barlow, 1978; Burgess, et al., 1981) employ the terms ‘external noise exclusion’ (equivalent to sampling efficiency) and ‘internal noise reduction’ (multiplicative internal noise). These terms imply an attentional change in sensitivity.

2 Predictions of the Sum of weighted likelihoods model were estimated using Monte Carlo simulations of 10,000 trials for each SNR.

3 Note that the argument that a tuning mechanism implies a limited capacity attentional process applies only to the current precueing task. A precueing task readily could be designed in which an ideal observer with unlimited capacity would also predict a tuning change. One example would be a task in which the signal is not known exactly, (or, in other words, the signal changes from trial to trial,) and the observer receives a precue on the probable signal identity (not location). See Eckstein, et al. (2002) for a discussion of this issue.

4 In this technique (Ahumada & Lovell, 1971; Ahumada, 1996; Beard & Ahumada, 1998), a task is presented in image noise, and the noise images are averaged within categories based on response outcomes. For example, in a yes/no task, the noise images are averaged separately for the hits, misses, false alarms, and correct rejections. The averaged noise images, or the classification images, can be interpreted as the perceptual template of the observer. In the case of the false alarm trials, the noise images must have had a random perturbation that led the observer to falsely judge the presence of the target. Thus, the classification image for the false alarms indicates the averaged noise field that the observer mistook as a signal.

5 This maximum decision rule forms the basis of the ‘spatial uncertainty’ model for visual search proposed by Palmer (Palmer, et al., 1993; Palmer, 1995; Palmer, et al., 2000) and others (e.g., Eckstein, et al., 2000).

6 Simulations of the maximum likelihood model were performed with 100,000 trials for each SNR.
References


