Computation of slant specified by cues

Slant from perspective convergence, part 1:
Construction of the projection \((x_1, y_1)\) on the screen (blue) of point \((x_0, y_0)\) of the rectangle (red) that is slanted by \(\phi\) about the vertical axis. 
\(zc\) is the center of projection.

\[
y_1 = -zc \times y_0 / (x_0 \times \sin[\phi] - zc);
\]

\[
x_1 = -zc \times x_0 \times \cos[\phi] / (x_0 \times \sin[\phi] - zc);
\]
Slant from perspective convergence, part 2:

Construction of point \((x_2, z_2)\) lying on the plane (red) defined by the unchanged height \((y_0)\) of the rectangle after that the screen has been slanted by \(\sigma\). 

\(z_v\) is the viewing point.

\[
\begin{align*}
z_p &= x_1 \times \sin(\sigma) - z_v; \\
\text{(* } z_p \text{ is the } z-\text{distance from the point on screen to } z_v \text{ *)} \\
x_p &= x_1 \times \cos(\sigma); \text{(* } x_p \text{ is the distance from the point on screen to the } z-\text{axis *)} \\
z_{pp} &= z_p \times y_0 / y_1; \text{(* } z_{pp} \text{ is the } z-\text{distance from } z_2 \text{ to } z_v \text{ *)} \\
z_2 &= z_{pp} + z_v; \\
x_2 &= x_p \times z_{pp} / z_p; \\
\upsilon &= \text{ArcTan}\left(\frac{z_2}{x_2}\right); \\
\text{Simplify}[\upsilon] \rightarrow zc \times r \text{(* } r \text{ is the ratio of } z_v \text{ to } zc \text{ *)} \\
\text{ArcTan}\left[\text{Tan}[\sigma] + r \text{ Sec}[\sigma] \text{ Tan}[\phi]\right]
\end{align*}
\]
The viewing point coincides with the center of projection: \( zv = zc \)

\[
\text{upsilon1} = \text{Simplify}[\text{upsilon}] /. \text{zv} / \text{zc} \rightarrow 1
\]

\[
\text{Plot3D}[\text{upsilon1}, \{\sigma, -\Pi/2, \Pi/2\}, \{\phi, -\Pi/2, \Pi/2\},
\quad \text{BoxRatios} \rightarrow \{1, 1, 1\}]
\]

\[
\text{ArcTan}[\text{Tan}[\sigma] + \text{Sec}[\sigma] \times \text{Tan}[\phi]]
\]
The viewing point is further away than the center of projection: \( z_v > z_c \).

\[
\upsilon_1 a = \text{Simplify}[\upsilon]/. z_v/z_c \to 10
\]

\[
\text{Plot3D}[\upsilon_1 a, \{\sigma, -\Pi/2, \Pi/2\}, \{\phi, -\Pi/2, \Pi/2\},
\quad \text{BaseStyle} \to \{21, \text{FontFamily} \to "Helvetica"\},
\quad \text{ColorFunction} \to \text{Function}[\{\sigma, \phi, \upsilon_1 a\},
\quad \text{ColorData}["RedBlueTones", "Reverse"]][\phi],
\quad \text{BoxRatios} \to \{1, 1, 1\}]
\]

\[
\text{ArcTan}[	ext{Tan}[\sigma] + 10 \text{Sec}[\sigma] \text{Tan}[\phi]]
\]
The viewing point is closer than the center of projection: $z_v \ll z_c$

$$\upsilon_{1b} = \text{Simplify}[\upsilon] / . z_v / z_c \to 0.1$$

Plot3D[\upsilon_{1b}, \{\sigma, -\Pi/2, \Pi/2\}, \{\phi, -\Pi/2, \Pi/2\},
   BaseStyle \to \{21, \text{FontFamily} \to \text{"Helvetica"}\},
   ColorFunction \to \text{Function}[\{\sigma, \phi, \upsilon_{1b}\},
   \text{ColorData}["RedBlueTones", "Reverse"]][\phi]],
   BoxRatios \to \{1, 1, 1\}]$

\text{ArcTan}[\tan[\sigma] + 0.1 \sec[\sigma] \tan[\phi]]$
Relationship between line inclination ($\alpha$), line separation ($\beta$) and slant from perspective convergence

$$
\beta = \ldots \; zv = \ldots \; xp = \ldots \\
p1 = \{xp, zv \tan[\beta] + xp \tan[\alpha], 0\}; \quad (\ast \text{point of a line at height } \beta \text{ on screen inclined by } \alpha \ast)
\\
p2 = \{0, 0, zv\}; \quad (\ast \text{viewing point } \ast)
\\
p3 = \{x, y, z\}; \quad (\ast \text{virtual point that is associated with } p1 \ast)
\\
c1 = (1 - t) p2 + t p1; \quad (\ast \text{p3 on the line between } p1 \text{ and } p2 \ast)
\\
c2 = zv \tan[\beta]; \quad (\ast \text{pictorial line is horizontal } \ast)
\\
e = \text{Eliminate}[p3 = c1 \&\& p3 = \{x, c2, z\}, \{t, y\}];
\\
s = \text{Solve}[e, \{x, z\}];
\\
\upsilon_{1c} = \text{ArcTan}[z / x] \div s
\\
\beta = 6 \text{ Degree}; \text{Plot}[\upsilon_{1c}, \{\alpha, -\pi / 2, \pi / 2\},
\\
\text{PlotRange} \rightarrow \{\{-\pi / 5, \pi / 5\}, \{-\pi / 2, \pi / 2\}\},
\\
\text{PlotStyle} \rightarrow \{\text{Black}, \text{Thick}\}, \text{AspectRatio} \rightarrow 1 / \text{GoldenRatio},
\\
\text{Ticks} \rightarrow \{\{-4 \beta, "-4\beta"\}, \{-2 \beta, "-2\beta"\}, \{2 \beta, "2\beta"\}, \{4 \beta, "4\beta"\}\},
\\
\{\{-\pi / 2, \pi / 2\}\}, \text{BaseStyle} \rightarrow \{21, \text{FontFamily} \rightarrow \"Helvetica\"\}]

\{\text{ArcTan}[\cot[\beta] \tan[\alpha]]\}
Slant from horizontal compression alone

\[ zv = \; \]
\[ \upsilon_{2p} = \text{ArcCos}[x_2 / x_0] \]
\[ \upsilon_{2m} = -\text{ArcCos}[x_2 / x_0] \]

Plot3D[{\upsilon_{2p}, \upsilon_{2m}}, \{\sigma, -\pi / 2, \pi / 2\},
\{\phi, -\pi / 2, \pi / 2\}, BaseStyle \to \{21, FontFamily \to "Helvetica"\},
ColorFunction \to \text{Function}[\{\sigma, \phi, \upsilon_{2p}\},
\text{ColorData}[\{"RedBlueTones", "Reverse"\}][\phi]],
BoxRatios \to \{1, 1, 1\}]
\text{ArcCos}[\cos[\sigma] \cos[\phi]]
-\text{ArcCos}[\cos[\sigma] \cos[\phi]]
Slant from horizontal compression disambiguated by the sign of perspective-specified slant

\[ \text{Supplementary material.nb} \]

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Slant from screen cues

\[ \upsilon_4 = \sigma \]

\[ \text{Plot3D}[\upsilon_4, \{\sigma, -\pi/2, \pi/2\}, \{\phi, -\pi/2, \pi/2\}, \]

\[ \text{BaseStyle} \rightarrow \{21, \text{FontFamily} \rightarrow "Helvetica"\}, \]

\[ \text{ColorFunction} \rightarrow \text{Function}[\{\sigma, \phi, \upsilon_4\}, \]

\[ \text{ColorData}["RedBlueTones", "Reverse"]][\phi], \]

\[ \text{BoxRatios} \rightarrow \{1, 1, 1\} ]\]