Computation of slant specified by cues

Slant from perspective convergence, part 1:
Construction of the projection \((x_1, y_1)\) on the screen (blue) of point \((x_0, y_0)\) of the rectangle (red) that is slanted by \(\phi\) about the vertical axis.
\(zc\) is the center of projection.

\[
y_1 = -zc \cdot y_0 / (x_0 \cdot \sin[\phi] - zc);
\]
\[
x_1 = -zc \cdot x_0 \cdot \cos[\phi] / (x_0 \cdot \sin[\phi] - zc);
\]
Slant from perspective convergence, part 2:
Construction of point \((x_2, z_2)\) lying on the plane (red) defined by the unchanged height \((y_0)\) of the rectangle after that the screen has been slanted by \(\sigma\).

\(z_v\) is the viewing point.

```
zp = x1 * Sin[\(\sigma\)] - zv;  (* zp is the z-distance from the point on screen to zv *)
xp = x1 * Cos[\(\sigma\)]; (* xp is the distance from the point on screen to the z-axis *)
zpp = zp * y0 / y1; (* zpp is the z-distance from z2 to zv *)
z2 = zpp + zv;
x2 = xp * zpp / zp;
upsilon = ArcTan[(z2) / x2];
Simplify[upsilon] /. zv -> zc * r (* r is the ratio of zv to zc *)
ArcTan[Tan[\(\sigma\)] + r Sec[\(\sigma\)] Tan[\(\phi\)]]
```
The viewing point coincides with the center of projection: \( zv = zc \)

\[
\text{upsilon1} = \text{Simplify}[\text{upsilon}] /. zv/ zc \rightarrow 1
\]

```
Plot3D[upsilon1, \{\sigma, -\Pi/2, \Pi/2\}, \{\phi, -\Pi/2, \Pi/2\},
  BaseStyle \rightarrow \{21, FontFamily \rightarrow \"Helvetica\"\},
  ColorFunction \rightarrow \text{Function}[\{\sigma, \phi, \text{upsilon1}\},
    \text{ColorData}[\{\"RedBlueTones\", \"Reverse\"\}][\phi]],
  BoxRatios \rightarrow \{1, 1, 1\}]
```

\[
\text{ArcTan}[\text{Tan}[\sigma] + \text{Sec}[\sigma] \text{Tan}[\phi]]
\]
The viewing point is further away than the center of projection: $z_v \gg z_c$.

upsilon1a = Simplify[upsilon] / . zv / zc -> 10
Plot3D[upsilon1a, \{\sigma, -\Pi/2, \Pi/2\}, \{\phi, -\Pi/2, \Pi/2\},
  BaseStyle -> {21, FontFamily -> "Helvetica"},
  ColorFunction -> Function[\{\sigma, \phi, upsilon1\},
    ColorData["RedBlueTones", "Reverse"]][\phi]],
  BoxRatios -> {1, 1, 1}]
ArcTan[Tan[\sigma] + 10 Sec[\sigma] Tan[\phi]]
The viewing point is closer than the center of projection: $z_v \ll z_c$

```math
upsilon1b = Simplify[upsilon] /. zv/zc \to 0.1
Plot3D[upsilon1b, \{\sigma, -\Pi/2, \Pi/2\}, \{\phi, -\Pi/2, \Pi/2\},
   BaseStyle \to \{21, FontFamily \to "Helvetica"\},
   ColorFunction \to Function[\{\sigma, \phi, upsilon1\},
   ColorData["RedBlueTones", "Reverse"]][\phi],
   BoxRatios \to \{1, 1, 1\}]
ArcTan[Tan[\sigma] + 0.1 Sec[\sigma] Tan[\phi]]
```
Relationship between line inclination ($\alpha$), line separation ($\beta$) and slant from perspective convergence

$$\beta = .; zv = .; xp = .;$$
$$p1 = \{xp, zv \tan[\beta] + xp \tan[\alpha], 0\};$$
(* point of a line at height $\beta$ on screen inclined by $\alpha$ *)
$$p2 = \{0, 0, zv\};$$
(* viewing point *)
$$p3 = \{x, y, z\};$$
(* virtual point that is associated with $p1$ *)
$$c1 = (1 - t) p2 + t p1;$$
(* $p3$ on the line between $p1$ and $p2$ *)
$$c2 = zv \tan[\beta];$$
(* pictorial line is horizontal *)
$$e = \text{Eliminate}[p3 = c1 \&\& p3 = \{x, c2, z\}, \{t, y\}];$$
$$s = \text{Solve}[e, \{x, z\}];$$
$$\upsilon_{\text{lc}} = \text{ArcTan}[z/x] /. s$$
$$\beta = 6 \text{ Degree};$$
$$\text{Plot}[\upsilon_{\text{lc}}, \{\alpha, -\pi/2, \pi/2\},$$
$$\text{PlotRange} \to \{\{-\pi/5, \pi/5\}, \{-\pi/2, \pi/2\}\},$$
$$\text{PlotStyle} \to \{\text{Black, Thick}\},$$
$$\text{AspectRatio} \to 1/\text{GoldenRatio},$$
$$\text{Ticks} \to \{\{-4 \beta, "-4\beta"\}, \{-2 \beta, "-2\beta"\}, \{2 \beta, "2\beta"\}, \{4 \beta, "4\beta"\}\},$$
$$\{-\pi/2, \pi/2\}\},$$
$$\text{BaseStyle} \to \{21, \text{FontFamily} \to "Helvetica"\}]$$

\{ArcTan[Cot[\beta] \tan[\alpha]]\}
Slant from horizontal compression alone

\[ zv = .; \]
\[ \upsilon_{2p} = \text{ArcCos}[x2/x0] \]
\[ \upsilon_{2m} = -\text{ArcCos}[x2/x0] \]
\[ \text{Plot3D}[\{\upsilon_{2p}, \upsilon_{2m}\}, \{\sigma, -\pi/2, \pi/2\}, \{\phi, -\pi/2, \pi/2\}, \text{BaseStyle} \rightarrow \{21, \text{FontFamily} \rightarrow \text{"Helvetica"}\}, \text{ColorFunction} \rightarrow \text{Function}[\{\sigma, \phi, \upsilon_{2p}\}, \text{ColorData}[\{"\text{RedBlueTones}\", \text{"Reverse"}\}][\phi]], \text{BoxRatios} \rightarrow \{1, 1, 1\}] \]

\[ \text{ArcCos}[\text{Cos}[\sigma] \text{Cos}[\phi]] \]
\[ -\text{ArcCos}[\text{Cos}[\sigma] \text{Cos}[\phi]] \]
Slant from horizontal compression disambiguated by the sign of perspective-specified slant

\[ zv = \text{Simplify}[\text{Sign}[\upsilon_1] \text{ArcCos}[x2/x0]] \]

\[
\text{Plot3D}[
\upsilon_3, 
\{\sigma, -\pi/2, \pi/2\}, \{\phi, -\pi/2, \pi/2\},
\text{BaseStyle} \rightarrow \{21, \text{FontFamily} \rightarrow "Helvetica"\},
\text{ColorFunction} \rightarrow \text{Function}[\{\sigma, \phi, \upsilon_3\},
\quad \text{ColorData}[\{"RedBlueTones", "Reverse"\}][\phi]],
\text{BoxRatios} \rightarrow \{1, 1, 1\}
\]

\[ \text{ArcCos}[\cos[\sigma] \cos[\phi]] \text{Sign}[\text{ArcTan}[\tan[\sigma] + \sec[\sigma] \tan[\phi]]] \]
Slant from screen cues

\[
\upsilon_4 = \sigma
\]

Plot3D[\(\upsilon_4, \{\sigma, -\pi/2, \pi/2\}, \{\phi, -\pi/2, \pi/2\}\),
BaseStyle \to \{21, FontFamily \to \"Helvetica\"\},
ColorFunction \to Function[\{\sigma, \phi, \upsilon_4\},
   ColorData[\"RedBlueTones\", \"Reverse\"]][\phi]],
BoxRatios \to \{1, 1, 1\}]

\(\sigma\)