Supplementary Methods and Results

Psychophysical experiments: General

Experimental participants and sessions

Psychophysical experiments were run on two members of the author’s research group (author S0, S1) who were experienced with the task, as well as on 7 naive participants (S2-S8) recruited from two undergraduate courses (PSB-4002, EXP-3202). All procedures were approved by the Institutional Review board (IRB) at Florida Gulf Coast University (FGCU Protocol ID: 2014-01), and all subjects provided written informed consent in accordance with the Declaration of Helsinki.

General experimental procedures

Subjects performed a single-interval 2AFC orientation discrimination task where contrast and orientation were varied in a factorial manner. Each trial was comprised of a test interval followed by a decision interval. During the test interval, a grating stimulus perturbed by $d\phi$ either clockwise or counter-clockwise (with equal probability) from vertical (orientation: $\pi/2 \pm d\phi$) was presented for 250 msec. After a 250 msec delay was the decision interval, where the subject was cued to indicate the perceived grating orientation with respect to vertical (counter-clockwise, clockwise) using a key-press (left or right arrow-key). Every 100 trials, subjects were allowed to rest for as long as they desired.

The task was performed at a viewing distance of 1 meter (using a chin rest), with stimulus size scaled to subtend 1 degree of visual angle (dva) at this distance. The grating stimuli had fixed spatial frequency of 4 cycles per degree (cpd) were presented at variable contrast $0 \leq c \leq 100\%$. Stimuli were presented on a 13-bit gamma corrected (gamma=1.0) Display++ CRT monitor (Cambridge Research LTD: http://www.crsltd.com/) having gray-scale (zero-contrast stimulus) illumination of 100 cd/m$^2$. Experiments were conducted in a dark windowless room (background illumination < 0.002 cd/m$^2$). Experimental con-
trol was managed by a Dell Optiplex 9020 Mini Tower (Intel® i7) running custom-authored software written in MATLAB® using routines from Psychtoolbox-3 (Brainard, 1997).

Psychophysical experiments: Paradigm details

Stimuli

The goal of Experiment 1 was to obtain psychophysical data to which we could fit and validate the model

\[ P(b = 1 \mid s, \theta, \omega) = \Phi \left( K \sqrt{\psi(c)} d\phi \right) \]  

(1)

and demonstrate that we are able to accurately infer the parameters \((n, c_{50})\) of the Naka-Rushton function

\[ \psi_1(c, \eta^{(1)}) = \frac{c^n}{c^n + c_{50}^n}, \]  

(2)

describing neuronal contrast gain.

A short version of Experiment 1 (800 trials) was performed on S0, S1 and a long version of Experiment 1 (2000+ trials over 2 testing days) was performed on S1-S8. For subject S1 (who performed both versions) all data presented is from the long version unless otherwise noted. In the short version, subjects S0 and S1 ran 800 trials with stimuli chosen at random (uniform probability, IID sampling with replacement) from a pre-defined 2-D factorial grid of contrast and orientation values. Contrast was varied logarithmically (base 2) over a mostly low-contrast range from 2 to 32 percent in 5 steps (2, 4, 8, 16, 32). Orientation \(d\phi\) was logarithmically spaced (base 10) from \(10^{-0.5}\) (0.32) to \(10^{0.65}\) (4.46) degrees, making this a challenging task. Grating phase was randomized for each trial to one of 10 evenly spaced values in \([0, 2\pi]\). No feedback was given. In the long version of Experiment 1, the orientation values \(d\phi\) were the same as above, but contrast was varied from 1 to 32 percent in 6 steps (1, 2, 4, 8, 16, 32). This stimulus grid is illustrated in Supplementary Fig. 2. In contrast to the short version of Experiment 1, there were an equal number of repetitions of each stimulus on the grid, as opposed to random sampling from the grid.
Experimental Sessions

Data collection was broken up over a series of two separate testing days so that at least 2000 total Experiment 1 trials could be collected from each subject (S1-S8). In a session prior to the two testing days, all naive subjects (S2-S8) were given one or two (S2, S4) practice sessions (at least 800 trials, data not analyzed) with auditory feedback. During the two testing sessions no feedback was given. Each testing session took roughly 1 hour, and subjects were allowed to rest every 100 trials.

On the first testing day, 1200 Experiment 1 trials were performed, and during the second testing day 840 Experiment 1 trials were performed (S2 performed 824). Due to possible effects of perceptual learning or other factors which may change performance across testing days, we our performed our model comparison and generalization (Supplementary Fig. 4) paradigms using model fits to the Experiment 1 data collected that same testing day. We ran our model comparison (Experiment 2 or C-phase) paradigm on testing day 1 using model parameter estimates attained from the 1200 Experiment 1 (E-phase) trials collected that day. On testing day 2, in addition to running additional (840) Experiment 1 trials, subjects S1-S8 also ran an additional 150 trials (Experiment 3) testing the ability of the model to generalize to novel stimuli \((c = 1, d \phi = 5, 6, 7, 8, 9:\) see Supplementary Fig. 2). For S1-S8 (but not S0), the model predictions shown in Supplementary Fig. 4 (blue lines) were attained from model fits to the Experiment 1 trials (840) collected during that same day (testing day 2).

Lapse Rates

When attaining parameter estimates, the mathematical form of the psychometric model was extended to include a lapse rate parameter \(\lambda\) (also estimated from data), so the final psychometric model was given by

\[
P(b = 1 \mid s = (d \phi, c)^T, K, \eta, \lambda) = \frac{\lambda}{2} + (1 - \lambda)\Phi \left( K \sqrt{\psi(c, \eta)}d \phi \right). \tag{3}\]
Previous studies have shown that inclusion of the lapse rate is often important for attaining accurate estimates of sensory thresholds (Wichmann and Hill, 2001; Prins, 2012). The values of estimated parameters as well as the lapse rates for each subject are given in Supplementary Tables 1, 2.

Model Comparison

We compared the fits of the model (3) with different assumptions about the form of $\psi(c, \eta)$ using the Akaike Information Criterion (AIC), an information-theoretic model selection technique which rewards good fits to data while penalizing model complexity (Akaike, 1974). This quantity is given by

$$AIC_i = -2 \ln L_i + 2d_i,$$  \hspace{1cm} (4)

where $L_i$ is the data likelihood under model $i$ and $d_i$ the number of parameters in model $i$. Following previous authors (Qamar et al., 2013), we present $-0.5 \cdot AIC$, so that the model with larger (less negative) value is preferred.

We also considered the Bayes Information Criterion (BIC), an asymptotic approximation to the likelihood of the observed psychophysical data $D_\psi$ which like the AIC rewards good fitting while automatically penalizing model complexity (Schwarz et al., 1978; Bishop, 2006). The BIC for model $i$ is given by

$$BIC_i = 2 \ln L_i - d_i \ln n,$$  \hspace{1cm} (5)

where $n$ is the number of observations. We present $0.5 \cdot BIC$ for direct comparison with $-0.5 \cdot AIC$. We see from (5) that for large $n$, the BIC penalizes model complexity more strongly than the AIC. We define a preference index $P_{i-j} = AIC_i - AIC_j$, with a positive value indicating a preference for model $i$ over $j$. When indicated, we define $P_{i-j}$ using the BIC (5) instead of the AIC (4).

In our analyses, model 1 was the model (1) with Naka-Rushton gain (2). model 2 assumed
Tanh gain

\[ \psi_2(c, \eta^{(2)}) = \tanh(bc) = \frac{e^{bc} - e^{-bc}}{e^{bc} + e^{-bc}}, \]  

having parameter \( \eta^{(2)} = b \). Model 3 permitted the possibility of non-monotonic Gaussian-shaped contrast tuning, and was defined by

\[ \psi_3(c, \eta^{(3)}) = \exp\left(-\frac{(c - \mu)^2}{2\sigma^2}\right), \]  

with parameter vector \( \eta^{(3)} = (\mu, \sigma)^T \).

References


Supplementary Tables

Supplementary tables.

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<thead>
<tr>
<th>Subject</th>
<th>trials</th>
<th>n</th>
<th>c50</th>
<th>λ</th>
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Table 1: Values of Naka-Rushton parameters, lapse rates and AICi estimated from Experiment 1 (short version).

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<tr>
<th>Subject</th>
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Table 2: Values of Naka-Rushton parameters, lapse rates and AICi estimated from Experiment 1 (long version).

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Table 3: Percent correct model choice for simulations of Experiment 2 (200 trials) with Naka-Rushton model as ground truth using OCS and IID stimuli.
Supplementary Figures

Figure 1: Residual sum-of-squares (SS) error for fits of the model (1) with Naka-Rushton (2) and Tanh (6) gains to the psychophysical threshold data in Fig. 3 (Skottum et al., 1987). Although the mean SS error is larger for (6) than (2), this effect is not significant (paired samples t-test, \( n = 6, p > 0.1 \)).

Figure 2: Stimulus set used in Experiment 1 to estimate the model (S1-S8). Right panel plots contrast values on a log (base 2) scale.
Figure 3: Same as Fig. 5 in main text, but for remaining subjects.
Figure 4: Observed subject performance (black diamonds) to a set of novel low-contrast stimuli \( (c = 1\%) \), along with predictions by the model (1) with Naka-Rushton (2) contrast tuning (blue line). Dotted blue lines denote 95% confidence intervals. We see reasonable agreement with the model for many (but not all) subjects.
Figure 5: Model preference $P_{1-2}$ for all subjects ($n = 9$) defined using the AIC and BIC, based on fits from Experiment 1 (2000+ trials). The BIC more severely penalizes model complexity than the AIC, changing the final preference for 1 subject.

Figure 6: Dynamics of model preference $P_{1-2}$ as a function of number of trials for Experiment 1 for the AIC (left) and BIC (right). Trials 200 – 2000 are plotted here (20 trial step) for the $n = 8$ subjects completing 2000+ trials.
Figure 7: Median changes in model preference ($\Delta P_{1-2}$) from 100 Monte Carlo simulations of Experiment 2 are correlated ($n = 9$, $r = 0.71$, $p = 0.03$) with the changes in model preference ($\Delta P_{1-2}$) actually observed in Experiment 2.

Figure 8: Results of $N_{mc} = 100$ Monte Carlo simulations of Experiment 2 (S0, S1) where the ground truth assumed Tanh (6) contrast tuning (model 2). We see that OCS stimuli (blue curves) were more effective than IID stimuli (green curves) in changing the model preference $P_{1-2}$ towards model 2 ($\Delta P_{1-2} < 0$). Thick line denote medians, thin lines delimit middle 95% of values.