Wide two-dimensional field laser ray-tracing aberrometer

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There is an increasing interest in measuring the peripheral optical quality of the eye. Optical aberrations have been studied extensively in the center of the visual field due to the development of Hartmann–Shack wavefront sensor. However, experimental data on the peripheral field of view are still scarce, partly due to the fact that this evaluation presents various challenges. Here, we propose a novel device based on the laser ray-tracing (LRT) aberrometer, which is well suited for measuring the off-axis aberrations. The proposed instrument is able to measure a wide (±40°) 2D visual field and is based on three main design principles: spiral-shaped sampling of the visual field, real-time detection of the eye’s entrance pupil, and automatic shaping and delivering of the ray bundle that optimally samples the eye pupil. We present experimental data obtained on 11 healthy subjects and a novel analysis based on a 2D quadratic model of the aberrations as a function of visual field and azimuth. The obtained results are consistent with previous findings.

Keywords: laser ray-tracing aberrometer, Hartmann–Shack wavefront sensor, visual field


Introduction

The optical aberrations of the human eye were extensively studied due to the development of Hartmann–Shack wavefront sensor (Liang, Grimm, Goelz, & Bille, 1994) and other methods of ocular aberrometry such as laser ray tracing (Navarro & Losada, 1997). Since then, many basic and clinical studies of ocular aberrations were published, but the great majority of them were restricted to the center of the visual field. In contrast, experimental data on peripheral optical quality are still scarce, despite the fact that the eye is a wide-angle optical system (Escudero-Sanz & Navarro, 1999). Initial studies were focused on peripheral refractive errors (Ferree, Rand, & Hardy, 1931; Rempt, Hoogerheide, & Hoogenboom, 1971). Later, measurements of the double-pass point spread function (PSF; Jennings & Charman, 1981) and the modulation transfer function (MTF; Navarro, Artal, & Williams, 1993) were performed across the visual field. To our knowledge, the first direct measurements of the monochromatic aberrations of the eye across the horizontal meridian were made using the laser ray-tracing method (Navarro, Moreno, & Dorronsoro, 1998). Similar measurements were reported using a Hartmann–Shack wavefront sensor (Atchison & Scott, 2002). Since then, different studies were reported, but most of them were limited to a few visual fields along the horizontal meridian (Lundström, Gustafsson, & Unsbo, 2009), or when they considered different meridians, then they were limited to the parafoveal region (Sheehan, Goncharov, O’Dwyer, Toal, & Dainty, 2007). Only recently, peripheral aberrations have been assessed across a two-dimensional (2D) and a relatively wide visual field of 20° × 15° (Mathur, Atchison, & Charman, 2009a, 2009b).

The growing number of studies in the literature demonstrates an increasing interest in the peripheral optical quality of the eye. However a two-dimensional scan of visual field is time consuming and aberration measurements at different visual angles and meridians present additional difficulties. For this reason, new instruments were developed to perform a fast scan of the horizontal meridian (Jaeken, Lundström, & Artal, 2011) and even scan a 2D (±15°) visual field (Wei & Thibos, 2010). Almost all of these instruments and studies are based on Hartmann–Shack (H–S) wavefront sensors. The main limitation of H–S sensors is that they use a monolithic lenslet array to sample the exit pupil. Then, the problem is that the array cannot adapt to the varying pupil shape (and size) and adjust its sensitivity to different levels of aberrations in real time (including changes in defocus and astigmatism with visual angle.) For wide fields, one may need to lower sensitivity in order to expand the dynamic range for avoiding saturation.
We believe that the laser ray-tracing (LRT) method (Navarro & Losada, 1997) is especially well suited to measure off-axis aberrations (Navarro et al., 1998). The main advantages are:

1. Retinal spots are recorded sequentially (i.e., independently from each other), thus avoiding potential cross talk when aberrations are large. This implies a higher dynamic range.

2. It permits to adapt the sampling pattern, in real time, allowing to span all the pupil area no matter its shape, size, position, or orientation. Additionally, the pattern can be tailored for satisfying different optimization criteria, for instance, non-redundant (spiral) sampling (Navarro, Arines, & Rivera, 2011).

3. One has full flexibility to deliver rays at any pupil position (polar coordinates radius and orientation: \(r, \theta\)) or field angle (spherical coordinates field and azimuth: \(\phi, \phi\)). This means that one has full freedom to implement different sampling patterns in both the pupil and the 2D visual field.

Here, we present a novel method to measure the monochromatic aberrations of the eye across a wide \((\pm 40^\circ)\) 2D visual field. There are two main differences with scanning H–S sensors (Jaeken et al., 2011; Wei & Thibos, 2010): Our system is based on the laser ray-tracing principle, and instead of having a mechanical (or opto-mechanical) device to scan the visual field, in our case the visual field is scanned by rotation of the eye. Furthermore, a pupil tracking system is used to find the position, size, and shape of the pupil and adapt (automatically and in real time) the ray bundle to optimally sample the pupil. In the following sections, we describe the method and present experimental results measured in a group of normal subjects.

## Methods

### Instrument design

The basic operation of the instrument is schematized in Figure 1. There are two main channels for probe (scanning channel) and measurement (measuring channel) plus two additional systems to control fixation (with on-axis and off-axis fixation targets) and for pupil tracking (CCD1). The LRT principle has been described before (Navarro & Losada, 1997). It consists of delivering a bundle of parallel narrow beams (rays), sequentially, to sample the pupil area. An infrared laser \((\lambda = 786 \text{ nm})\) with collimating optics emits a narrow Gaussian beam (width \(0.75 \text{ mm}\)). This beam impinges a 2D mirror scanner, placed in the focal plane of the collimating lens \(L_c\). This configuration produces a (sequential) bundle of parallel beams, which after reflection in beam splitter (BS1), impinge the eye at different pupil coordinates \((x, y)\). The power of the beam at the cornea was set between 6 \(\mu\)W and 10 \(\mu\)W. It was adjusted to match the dynamic range of the CCD camera for each subject. The chief beam going through the center of the pupil \((0, 0)\) reaches a certain point \(O\) on the retina, forming an approximately Gaussian spot. Due to aberrations in the eye, an arbitrary ray going through the pupil at coordinates \((x_A, y_A)\) deviates from its ideal trajectory and reaches the retina on a different point \(A\), forming another spot. The detection channel, composed by lenses \(L_1, L_2,\) and \(L_3\), images the impacts at \(O\) and \(A\) to points \(O'\) and \(A'\) on the CCD2, respectively. \(L_1\) and \(L_2\) form a Badal system, which images the eye's pupil on lens \(L_3\) and eventually permits compensating defocus. The
transverse ray aberration at \((x_A, y_A)\) is proportional to the distance between \(A'\) and \(O'\). By delivering a bundle of parallel rays through different pupil coordinates (i.e., sampling the pupil area), one obtains a sequence of spot images. These images are analyzed to compute the centroid of each spot, which in turn form the initial raw data set. They can be used to plot the spot diagram (set of transverse aberrations), or upon numerical integration, these raw data allow obtaining the wave aberration of the eye. Similarly, the aberrations can be assessed for arbitrary field angles by tilting the visual axis of the eye with respect to the sampling beam bundle (Navarro et al., 1998).

There are three main novel design strategies in the present instrument. First, the 2D visual field is sampled along a spiral line. As we discuss below, spirals provide an efficient way of sampling 2D domains, since they allow avoiding redundancy and simplifying the 2D sampling, since the spiral is a curved 1D line. Second, the instrument includes a pupil tracking and analysis subsystem, which permit to obtain the position, size, and shape of the pupil in real time. Third, the pupil-sampling pattern of rays (bundle) is adapted in real time to fit the pupil area for each field position. These combined features provide a rapid and versatile device to measure the aberrations of the eye across a 2D wide field.

**Spiral sampling of visual field**

The visual field is sampled along an exponential spiral line, in such a way that the sampling points lie along four curved branches as shown in Figure 2. The pattern is centered at the visual axis, so that both field angle \(\theta\) and azimuth (meridian) \(\phi\) are zero at the fovea: \(\theta_1 = \phi_1 = 0\). For off-axis points, the azimuth is sampled linearly (homogeneous), whereas field angle is an exponential function of the azimuth (inhomogeneous):

\[
\phi_i = i\Delta\phi, \quad (1a)
\]

\[
\theta_i = te^{b(\phi_i - \phi_0)}. \quad (1b)
\]

In the current version of the instrument, we have set the parameters \(\Delta\phi = 1.6535\) rad, \(t = 0.0873\) rad, \(\phi_0 = 1.6535\) rad, and \(b = 0.0699\) rad\(^{-1}\) to obtain 21 sampling points (1 on-axis and 20 off-axis) for the left eye, as shown in Figure 2. The resulting pattern covers a 2D visual field of nearly 80° wide and is evenly distributed along azimuths (meridians) and field angles. The off-axis points are grouped into four curved branches, which we labeled as superior (S), inferior (I), nasal (N), and temporal (T). Note that the frame of reference corresponds to the field in the retina.

There are several reasons to choose this type of pattern rather than square (Mathur et al., 2009b) or polar (Wei & Thibos, 2010) or similar ones. As it was mentioned above, one practical reason is that the spiral is a line (one-dimensional in nature), as opposed to these 2D sampling grids. One can unroll the spiral to obtain 1D plots of the two angular coordinates simultaneously (see Figures 6 and 7). Second, the spiral permits to avoid redundancy so that field and azimuth angles are never repeated. This implies a more efficient sampling in the sense of getting maximum (non-redundant) information with the minimum number of samples. Furthermore, in wavefront sensing, non-redundant (pupil) sampling spirals permit to recover double number of Zernike modes than standard grids (Navarro et al., 2011). Finally, another important reason for using an inhomogeneous exponential sampling of field angle is to match the cortical magnification factor of the visual system (Rovamo & Virsu, 1979). This magnification factor means that the receptive fields of cortical neurons scale (increase) nearly exponentially with visual field. The exponential spiral provides a good match with the nearly exponential decline of visual resolution with field angle.

An array of yellow light-emitting diodes (LEDs) was used as fixation targets to measure the peripheral field. These LEDs were fixed to the inner surface of a half-sphere of radius 250 mm with a central hole, as schematized in Figure 1. They were independently activated to scan the spiral in any previously chosen sequence. The on-axis fixation was viewed through the central hole, beam splitters BS3 and BS1, and collimator Lc. In this case, a cross-like target was used instead of a plain LED to enhance fixation. The subject eye (and eventually both eye and head for large field angles) rotates according to the visual field and azimuth of each fixation target. A bite bar with the imprint of each subject’s mouth is used to center the eye to the system, which also provides further stability and repeatability throughout different series of measurements.

![Figure 2. Spiral sampling pattern of the visual field used for the left eye. All the labels are in degrees. The frame of reference corresponds to the field in the retina. Samples can be grouped in four branches: superior (S), inferior (I), nasal (N), and temporal (T).](https://jov.arvojournals.org/pdfaccess.ashx?url=/data/journals/jov/932802/ on 11/24/2018)
Pupil tracking and fitting

An essential component of this instrument is the pupil analyzer (and tracker), which provides the position, size, and shape of the pupil, which is then used by the laser scanner to adapt the ray bundle to the pupil in real time. The pupil is illuminated by a ring of near infrared LEDs fixed to the inner surface of the half-sphere fixation support. The iris is imaged on the CCD1 (pupil camera) through lenses L1 and L4 and beam splitters BS1 and BS2. The analysis software (implemented in visual C++) basically consists of a histogram-based thresholding to obtain a binary image, an edge detection to find the pupil contour, and an ellipse fitting of that contour. It is worth remarking that the pupil of the eye is not circular even on axis, as the pupillary axis forms and angle \( \kappa \) with the visual axis. The pupil of the eye can have different shapes, but the elliptical fitting provides a reasonable approximation, significantly better than a circle, both on-axis and off-axis. The pupil analysis provides an ellipse defined by five parameters: the coordinates of its center \((x_0, y_0)\); the two major and minor semi-axes \(s_x, s_y\) and its orientation \(\phi_e\).

To program the laser scanner, for each measurement (pupil scan at a given field position), the system departs from a pre-calculated set of coordinates \(x_j, y_j\) corresponding to a sampling grid on a circle of unit radius (i.e., normalized). In the implementation used here, this set (schematized in Figure 3, left) forms a hexagonal grid of 37 samples \((j = 1, \ldots, 37)\). The actual coordinates sent to the scanner (Figure 3, right) are computed by applying an affine transform to all points in the grid. In matrix–vector notation:

\[
    x_j = R_{\phi_e}^{-1} S \Phi_{\phi_e} x_j^0 + x_0, \tag{2}
\]

where the different \(x_j\)s are column vectors of coordinates, \(R\) is the \((2 \times 2)\) rotation matrix around \(\phi_e\), and \(S\) is the scaling operator, i.e., a diagonal matrix whose elements are the two semi-axes \(s_x, s_y\). This affine transform is general, so that it can be applied to any type of pre-computed sampling grid on a circle. Figure 4 shows the elliptical fit to the pupil edge (dashed green line) and the sampling pattern (red dots) for the central and four more eccentric fixations.

Another strategy used in the present study is to scan a smaller pupil area, in order to get a more homogeneous pupil across visual field. In this case, the sampled elliptical area of the pupil is computed assuming the nominal affine transform due to perspective (Wei & Thibos, 2008). Thus, the rotation and scaling matrices are now functions of field coordinates: azimuth \(\phi_s = \phi_i - \pi/2\), that is, \(R_{\phi_s}\) and angle \(\theta_i\) so that the major and minor semi-axes in \(S\) will be given by \(s\) and \(s \cos(\theta_i)\):

\[
    R_{\phi_s} = \begin{pmatrix} \sin \phi_i & -\cos \phi_i \\ \cos \phi_i & \sin \phi_i \end{pmatrix}
    \quad \text{and} \quad
    S_{\phi} = s \begin{pmatrix} 1 & 0 \\ 0 & \cos \theta_i \end{pmatrix}, \tag{3}
\]

where \(s\) is a nominal pupil radius computed as the minimum of \(s_x\) and \(s_y \cos(\theta_i)\) to guarantee that the sampled ellipse fits inside the real pupil. This strategy permits to get the same eccentricity and orientation of the ellipse for different subjects, which facilitates comparisons and statistical analysis.

For each fixation, the pupil analysis and the computation of the coordinates of the ray bundle is performed at near video rate (25 Hz). Then, a laser shutter is opened and the scanner delivers the bundle of rays sequentially, synchronized with the retina camera CCD2. The maximum frame rate for the Gigabit Ethernet bus camera is approximately 200 Hz, but the actual operating frequency is somewhat slower to guarantee a good synchronism with the scanner. As a result, the acquisition of the 37 retinal spot images takes half a second. Overall, the system can work in a fast full automatic mode or in a slower supervised version, in which the experimenter verifies fixation and pupil fit and then clicks the mouse to perform a pupil scan. The automatic mode can complete the visual field scan in less than 1/2 min but is less reliable, especially with untrained subjects. In this study, we used the supervised mode, which typically takes between 5 and 10 min, for maximizing reliability.

Data analysis

The transverse and wave aberrations are computed for each field position following standard procedures that were described before (Navarro et al., 1998). The image of each retina spot is analyzed to obtain the corresponding centroid coordinates, leading to the transverse aberrations \((\Delta x_j, \Delta y_j)\). The complete set of coordinates can be displayed as a spot diagram. The wavefront reconstruction is obtained by the standard method of fitting these data to...
the partial derivatives of Zernike polynomials (ZPs) up to the 7th order to obtain the (35) coefficients of the ZP wavefront expansion. ZPs are defined and ordered according to the ANSI Z80.28 standard for reporting the optical aberrations of eyes.

A relevant issue is that for elliptical pupils, Zernike polynomials are not orthogonal. To solve this problem, the same method used before for off-axis LRT measurements (Navarro et al., 1998) was implemented. It basically consists of applying the inverse affine transform to recover the theoretical sampling pattern over the circle of unit radius $x_j^h$:

$$x_j^h = R_{\phi_s}^{-1} S^{-1} R_{\phi_s} (x_j - x_0). \tag{4}$$

In this way, the wave aberration is reconstructed over a unitary circular pupil for all subjects and field angles. Note that the pupil analysis provides a scaling factor $s$ that is the radius of the circle effectively sampled. Thus, the wavefront gradient is given by $\nabla W_j = s \Delta x_j$, where $\Delta x_j = (\Delta x_j, \Delta y_j)^T$, which is the transverse aberration described above. This provides a unified, normalized procedure to reconstruct the wavefront. The final step to obtain the true wave aberration is to apply again the warping given by the affine transform of Equation 2 (now ignoring the displacement $x_0$) to the reconstructed wavefront. It is important to notice that the warping affects the values of Zernike coefficients (Bara, Arines, Ares, & Prado, 2006), but the values of the wavefront do not change. In fact, metrics such as the RMS wavefront error, peak-to-valley difference, etc., are invariant under that affine warping.

The modal analysis usually performed on-axis (i.e., circular pupils) becomes tricky for off-axis measurements. While it is possible to recompute the coefficients for the Zernike polynomials on elliptical pupils (Bara et al., 2006; Lundström & Unsbo, 2007), it turns out that they loose orthogonality and hence do not provide a true modal description of the wavefront. However, the direct-inverse warping procedure described above does permit to define warped versions of Zernike modes (i.e., warped coma, etc.) by simply applying Equation 2 to each Zernike mode. By applying the inverse warping (Equation 4), then one passes again to the unitary circle, which permits to report wave aberrations in a normalized canonical way, invariant to pupil size, shape, and position. To complete that report, one needs to know the parameters of the affine transform (Equation 2), namely, $s$, $\theta$, $\phi$ (and $x_0$, $y_0$ when needed) to pass from that canonical representation to the actual physical pupil. For this reason, to ease the comparison

Figure 4. Pupil images for the central and four more eccentric field angles in one subject. The dashed green line shows the fit to the pupil edge and the red dots represent the center positions of the beams delivered by the scanner.
between different subjects, we assume the nominal affine transform defined by Equations 2 and 3. Here, the modal analysis will be made in terms of coefficients of warped canonical Zernike modes (WCZMs), i.e., referred to the unitary circle. In addition to its generality, the WCZM representation does not require implementing especial algorithms (Bara et al., 2006; Lundström & Unsbo, 2007) to recomputed the coefficients.

Calibrations

The system was calibrated on axis with an artificial eye consisting of an achromatic lens doublet, \( f' = 50 \text{ mm} \), corrected for spherical aberration, and a white screen placed on its focal plane. Different amounts of defocus and astigmatism were introduced by placing trial lenses in front of the artificial eye. The range covered by the trial lenses was from \(-4 \text{ D} \) to \(+4 \text{ D} \), in 1-D steps, both for spherical (S) and cylindrical (C) lenses. The measurements were repeated four times for each trial lens and linear regressions of measured \( (S_n \text{ and } C_n) \) versus nominal \( (S_n \text{ and } C_n) \) values were performed. The resulting linear fit was \( S_m = 1.011S_n + 0.05333 \ (R^2 = 0.9995) \) and \( C_m = 1.0C_n + 0.02889 \ (R^2 = 0.9989) \), both in diopters. This result suggests a high linearity and accuracy for the wide range (8 D) analyzed so far.

Results

Measurements were taken in a group of 15 volunteers between 23 and 57 years old, with an average of 33 years. The refractive errors were low or moderate, and sphere ranged between \(-2.5 \text{ D} \) and \(+0.5 \text{ D} \), with an average of \(-0.6 \text{ D} \). Cylinder ranged between \(-2 \text{ D} \) and 0 D, with an average of \(-0.4 \text{ D} \). The viewing conditions were near vision and natural pupil. The stimulus vergence was 4 D and only the left eye was measured in each subject. The subject head was fixed by means of a bite bar to optimize stability and repeatability throughout the various series of measurements. Each measurement, performed in semiautomatic (supervised) modality, was repeated four times or five times in some cases. A complete session, including subject alignment and 84–105 wavefront measurements (21 fields times 4 or 5 repetitions), lasted half an hour approximately. After data processing, subject data were considered for further analysis only if the set was complete (reliable measurements for all field positions) and if the pupil was equal or greater than 4-mm diameter. Consequently, we discarded data for 4 subjects, so the following results correspond to a group of 11 left eyes. The average pupil diameter for these 11 eyes and 21 field angles was 5.3 mm approximately. Nevertheless, the visual field analysis was made for the minimum common pupil diameter of 4 mm. The Zernike coefficients were computed for that common pupil using the method of Schwiegerling (2002).

Figures 5 and 6 show some examples of the results for individual eyes. Figure 5 represents the wave aberration maps, higher order aberration (HOA) only, for subject JP. Figure 6 shows the RMS HOA plotted against visual angle (\( \theta \)). Note that the spacing between points in the curve increases (exponentially) with \( \theta \). On the other hand, the spacing between points in terms of meridian (azimuth) is constant, \( \Delta \phi = 94.74^\circ \). This is one important advantage of spiral sampling. Since spiral is a line, one can unroll the spiral and plot the data against either \( \theta \) or \( \phi \). The four branches of the spiral, nasal, superior, temporal, and inferior are labeled with different colors (black, green, blue, and pink, respectively). The central panel corresponds to the same subject (JP) of Figure 5, who is a highly representative one. In Figure 5, we can observe the warping of the pupil that becomes apparent for field angles \( \theta > 20^\circ \). Another feature observed in Figure 5, that is also common to all subjects, is the dominance of coma for peripheral fields. The three examples of Figure 6 cover the main aspects that we found in our group of subjects. First of all, we can see a high intersubject variability. The first subject (left panel), CL, is an example of low HOA and a good homogeneity between the different branches of the spiral. The right panel (subject RN) represents the opposite case with higher values of HOA and large differences between branches. The central panel (subject JP) represents an intermediate case. As it will be further discussed below, HOAs tend to increase with field angle,

![Figure 5. Higher order aberrations (wavefronts) across the 2D visual field for subject JP (left eye).](Downloaded From: https://jov.arvojournals.org/pdfaccess.ashx?url=/data/journals/jov/932802/ on 11/24/2018)
which is consistent with previous findings (Atchison & Scott, 2002; Navarro et al., 1998). However, such increase is not monotonic and often shows remarkable differences among branches. An interesting feature, that is patent in these three examples (and we found the same behavior for most of the subjects), is that the minimum RMS value is not placed at the fovea ($E = 0^\circ$), but it is displaced between $5^\circ$ and $8^\circ$ depending on the subject. This suggests that the axis of best optical quality (minimum RMS HOA) may be closer to the optical axis ($\theta = 5^\circ$ nasal) than to the visual axis. This is patent for subjects CL and JP (and three more subjects). For several subjects (RN and others), however, the minimum is placed on the temporal retina. In summary, the axis of best optical quality is different from the visual axis for all the measured subjects, and for about 45% of subjects that axis, it seems to be close to the optical axis.

The following figures and data correspond to the average of 11 (left) eyes. In order to analyze the 2D distribution of the different aberrations, we performed least squares fit on the different average aberrations to a 2D polynomial of the form:

\[
\text{RMS} = a_{00} + a_{10}u + a_{01}v + a_{20}u^2 + a_{11}uv + a_{02}v^2 + \ldots,
\]

(5)

where RMS may correspond to any type of aberration, including the sum of all contributions (total) or to HOA, and the variables are $u = \theta\cos\phi$ and $v = \theta\sin\phi$. The RMS value for a given type of aberration is obtained as

\[
\text{RMS}_n^{[m]} = \sqrt{c_n^m + c_{-n}^{-m}},
\]

where $C$ is the Zernike coefficient. We have tested different degrees of polynomials and experienced that the second-degree approximation (Equation 5) provided the best balance between goodness of fit, reliability, and a simple physical interpretation of the resulting fit. In fact, for the second-order polynomial, the iso-RMS curves are a family of concentric ellipses (conic curves in general) defined by a common

<table>
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<tr>
<th>Total</th>
<th>HOA</th>
<th>Defocus</th>
<th>Astigmatism</th>
<th>Coma</th>
<th>Trefoil</th>
<th>Spherical aberration</th>
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<td>$a_{20}$</td>
<td>1.0766</td>
<td>0.3259</td>
<td>0.0779</td>
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<tr>
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<td>2.5913</td>
<td>0.4156</td>
<td>0.124</td>
</tr>
<tr>
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<td>0.0083</td>
<td>0.1395</td>
<td>0.3105</td>
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<tr>
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<td>0.57</td>
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<tr>
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<tr>
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<td>0.8928</td>
<td>0.61</td>
<td>0.9721</td>
<td>0.8529</td>
<td>0.7368</td>
</tr>
</tbody>
</table>

Table 1. Top: Fit parameters ($a_{ij}$) of Equation 5 on various aberrations and groups of aberrations. $R^2$ is the correlation coefficient of determination. Bottom: ($u_0, v_0$) are the coordinates of the minimum corresponding to each fitted polynomial. $Q$ and $\alpha$ are, respectively, the corresponding conic constant and the tilt angle.

Figure 6. RMS higher order aberrations of the left eyes of subjects CL, JP, and RN, as a function of field angle $\theta$. The four branches N, S, T, and I are labeled with different colors (black, green, blue, and pink). The bars indicate the standard deviation of the mean.
center \((u_0, v_0)\), elongation (conic constant \(Q\)), and orientation \(\alpha\) of the major axis. These parameters as well as the coefficients \(a_{ij}\) are listed in Table 1 for different low- and high-order aberrations.

Figures 7a–7d represent the average RMS values for total wavefront error, defocus, astigmatism, and HOA, respectively. Error bars here represent intersubject variability.

In these four figures, the red line represents the best fit quadratic polynomial (Equation 5). The fit is reasonable in general and especially good in the case of astigmatism. This simple quadratic model is powerful enough, since it allows us to explain the strong asymmetry between spiral branches. On the one hand, the model (red line) shows a strong non-monotonic behavior since it connects the points of the different spiral branches. On the other hand, if we observe the contour lines (iso-RMS) of the 2D quadratic models obtained for these RMS values (see Figure 8 for defocus, astigmatism, and HOA), they not only look smooth but are also monotonic around their centers. The center corresponds to the minimum RMS value in each case. This means that most of the asymmetry found between the four spiral branches can be explained by the fact that the center of symmetry for each particular aberration (or for total or HOA RMS) is off-axis. In these 2D contour plots, the visual axis is marked as a blue star cross, whereas the center of the contours is plotted as a red cross star. The quadratic model also reveals a lack of rotational symmetry, especially outstanding for defocus. The elliptical contours are strongly elongated, which suggest that defocus is much more uniform throughout the horizontal than along the vertical. Additionally, most of the gradient appears in the superior retina. This is consistent with the higher values obtained for the superior branch (green line in Figure 7b), while the other branches tend to be flat. Nevertheless, for defocus we obtained the poorest fit \((R^2 = 0.61)\), and hence, the quadratic approximation is not good in this case. The best fit was obtained for astigmatism \((R^2 = 0.97)\), followed by total and HOA. The goodness of fit was still reasonable \((R^2 = 0.85)\) for coma and spherical aberration but was poor for trefoil and defocus.

The relative contributions of the different aberrations to the total wavefront error are compared in Figure 9. The error bars are omitted for the sake of clarity. The main contribution to the total wavefront error (upper black line) is defocus (pink), especially on axis. Its contribution does not change much with field angle (but it changes with azimuth). Astigmatism (blue line) is the second contribution, which shows a quadratic increase (as expected from Seidel theory), in such a way that it becomes the main contribution for large field angles (\(\theta > 35^\circ\)). The third
contribution is due to HOA (red line), especially coma (narrow black line), which accounts for most of the HOA RMS. Trefoil has a still noticeable contribution, while spherical aberration (SA) shows low positive values at the fovea but with a negative slope so that they tend to be more negative toward the periphery. The low and even negative values of SA can be explained by accommodation since the stimulus vergence was 4 D (Lopez-Gil & Fernandez-Sanchez, 2010).

As we said above, our results suggest that the axis of best optical quality is different from the visual axis. If we leave defocus apart, as its field distribution might be dominated by retinal shape, it is patent that the two main contributions to peripheral wavefront error are astigmatism and HOA. The axis of minimum astigmatism is placed $-4.9^\circ$ (nasal) and $-1.1^\circ$ (inferior), which is surprisingly close to the average angle of the optical axis (5° nasal, ~2° inferior; Le Grand & El Hage, 1980). For the axis of minimum HOA, we obtained 1.4° nasal, 2.2° inferior. Each aberration (or group of aberrations) has its own axis of minimum RMS value, but (see Table 1) the angles of these axes with the visual axis are negative (nasal, inferior retina) and are not far from the average value of the optical axis.

Finally, our quadratic fit reveals a lack of rotational symmetry in the field distribution of main aberrations (astigmatism, HOA, coma, and spherical aberration). Contour lines show a clear elongation (see conic constants $Q$ in Table 1) with values between 0.35 and 0.6 that correspond to ellipsoids. The main axis of the ellipsoid (line of maximum elongation) with respect to the horizontal axis is 30.7° for HOA and slightly higher (33°–34°) for the two main off-axis aberrations, astigmatism and coma.

**Discussion**

A novel method for measuring wavefront aberrations over a wide two-dimensional (~80° diameter) visual field is presented. The device, based on laser ray tracing, combines several subsystems and design strategies to enhance performance. The non-redundant spiral sampling, of both field angle and azimuth (meridian), permits us to assess a wide 2D field with a reduced number of 21 samples. On the other hand, video-rate pupil tracking and analysis allows real-time alignment, scaling, and reshaping of the beam bundle to match position, size, and apparent shape of the pupil. In this way, the measuring...
system does not need to move (except for the two mirrors of the laser scanner), thus avoiding the need for system realignments, what in turn speeds up critically the whole measurement. The system is also well suited to measure in natural viewing conditions due to its high flexibility, which permits full and fast adaptation to pupil size, shape, and position. Here, we have assumed the standard approximation of considering an elliptical pupil that squeezes linearly with the cosine of the field angle. For the field angles considered here, this approximation to the actual pupil shape seems good enough. Nevertheless, our method could be generalized to consider more sophisticated models of the pupil shape (Fedtke, Manns, & Ho, 2010). Calibrations with an artificial eye showed a highly linear response over a broad dynamic range. This is a crucial property for measuring aberrations throughout a wide field, where the magnitude of aberrations can change dramatically from the center to the periphery.

As mentioned above, the spiral pattern used to sample the field of view is made of only 21 points. The choice of such a sparse sampling pattern may prevent us from detecting some features, as, for example, multiple minima across the field, which are expected for some aberration types when studying misaligned systems with no rotational symmetry. For example, astigmatism might show two minima (Espinosa, Mas, & Kasprzak, 2009). Another example is that since our spiral does not cover the area of the blind spot, then we do not observe defocus peaks. By contrast, the chosen pattern allows us to measure a wide 2D visual field in a very short time.

The aberrations for different field angles are measured by rotating the eye toward a set of stimuli. It has been reported (Ghosh, Collins, Read, Davis, & Iskander, 2011; Prado et al., 2009) that the rotation of the eye may yield small changes in some aberrations as defocus, primary astigmatism, and coma. With this in mind, the rotation of the whole head (as opposed to eye-only rotation) may render more accurate measurements. However, its implementation complicates considerably the system and slows significantly the measuring procedure. The eye rotation approach, as implemented here, provides a fair trade-off between accuracy and simplicity of operation.

Our experimental results are totally consistent with previous findings, taking into account the viewing conditions in the present study: left eyes, near vision (4 D stimulus vergence), analysis of wavefronts within a central 4 mm diameter natural pupil. Many authors reported a significant degree of symmetry between the wavefronts in left and right eyes (Castejon-Mochon, Lopez-Gil, Benito, & Artal, 2002; Liang & Williams, 1997; Porter, Guirao, Cox, & Williams, 2001). The main effect of accommodation associated to the near vision target was the reduction (even taking negative values) of spherical aberration (4th order) in the periphery. This is fully consistent with previous findings by Mathur et al. (2009b), who reported that there was little change in wave aberrations with accommodation, except for 4th-order SA that became more negative at all field locations. This change of SA toward more negative values with accommodation is a well-known effect for central vision (Lopez-Gil & Fernandez-Sanchez, 2010). According to Mathur et al., the rest of HOA and even astigmatism are basically independent of accommodation at any field angle.

### Agreement with Seidel theory

Both experimental data (Mathur, Atchison, & Scott, 2008) and eye models (Escudero-Sanz & Navarro, 1999) suggest that in the peripheral visual field wave aberrations follow the third-order Seidel theory of aberrations. Seidel coefficients basically depend on pupil radius and field angle ($\theta$). Fourth-order spherical aberration should be constant, that is, independent of $\theta$. Third-order aberrations (coma) should be linear with $\theta$, and second-order astigmatism and field curvature (defocus) should have a quadratic increase (in magnitude) with field angle. Previous studies (Mathur et al., 2008; Navarro et al., 1998) found a nearly linear dependency of the HOA RMS with field angle. Third-order aberrations, mainly coma, are the main contribution to HOA in the periphery, which is consistent with third-order optics (Seidel) approach. For astigmatism, the agreement with Seidel theory is patent, as astigmatism might show a quadratic increase with field (Lotmar & Lotmar, 1974; Rempit et al., 1971). In the study of Mathur et al. (2008), they obtained a field distribution of aberrations similar to the field dependence predicted by Seidel theory, except for defocus (field curvature). We obtained fully consistent results. In the peripheral visual field, most of the total wavefront RMS error comes from three contributions: defocus (which is the combined contribution of on-axis defocus, about 1 $\mu$m hyperopic defocus average across subjects, plus field curvature in Seidel theory of aberrations), astigmatism, and coma. Spherical aberration has a small contribution for near vision; trefoil has a measurable magnitude but much smaller than these main contributions. Higher orders show a little contribution to the RMS error for a 4 mm pupil.

Astigmatism shows a quadratic increase with field angle, while coma has a nearly linear dependency. The poor goodness of fit to the quadratic model obtained for defocus indicates a strong discrepancy with the Seidel theory in this case. This result is again consistent with previous studies. Most authors agree to explain such discrepancy by the shape of the retinal surface. Our results suggest that the image (field) curvature is similar to that of the retina, which would explain (see Figures 7b and 8a) that defocus is relatively homogeneous, except for the upper retina (green line in Figure 7b). Perhaps this could be explained by the possible deformation of the eye globe (and hence of the retina) under the action of gravity. Since gravity would pull down, the expected deformation of the retinal surface would cause a hyperopic shift in the upper
retina and the opposite effect in the lower retina. This would explain the distribution that we can observe in Figure 8a, namely, a displacement of the center downward (~26.4°), whereas the strong nearly horizontal elongation of the contour lines indicates much higher homogeneity along the horizontal direction. This would be in agreement with the low-field myopia phenomenon, observed in animal models (Hodos & Erichsen, 1990). These authors hypothesized that this could be an adaptation to keep the ground (closer) in focus. On the other hand, our spiral sampling of left eyes does not include points within the area of the blind spot, which explains why we do not observe defocus peaks (Mathur et al., 2008).

Our 2D analysis based on polynomial fit of the RMS contributions of different aberrations (or combinations of aberrations) revealed further features. First of all, each type of aberration has its own axis of best optical quality (BOQ) or minimum RMS. Interestingly enough, the BOQ axis obtained for (subject average) astigmatism was surprisingly close to the average optical axis. For HOA, that axis was placed at an intermediate location between the visual and optical axes (the axes obtained for coma and spherical aberration were similar). It is important to note that the agreement with the Seidel theory is true only if we consider this BOQ axis for these (dominant) aberrations. If instead of BOQ axis we consider the visual axis, we observe a non-monotonic dependence with field angle (see Figures 7c and 7d), which would be totally inconsistent with Seidel theory.

Possibly, the most relevant inconsistency of our data with Seidel theory revealed by the 2D contour plots is the lack of rotational symmetry. The field distributions of the most significant aberrations (astigmatism, coma, HOA RMS) show a clear elongation. Contour lines are elliptical with the major axis oriented ~33° with respect to the horizontal. The conic constants ~0.5 show a clear departure from the circular shape. The high consistency between the main off-axis aberrations (astigmatism, coma, and all HOAs), in their field distribution (center not far from the optical axis, conic constant, and orientation), suggests that this elliptical distribution of aberrations could be a real feature of the optical system of the eye. This cannot be explained by the third-order Seidel theory, unless one admits the possibility of applying an affine 2D warping of the image domain. Perhaps, such lack of rotational symmetry could be due to the interaction between on-axis and off-axis aberrations (especially coma and astigmatism). However, it should be mentioned that the sparse sampling toward the periphery given by the spiral could be inducing some bias to the quadratic fit. To test this hypothesis, we conducted a Monte Carlo computer simulation, applying the spiral sampling to a rotationally symmetric quadratic distribution. When these input data were free from noise, the result was unbiased (we obtained the original rotationally symmetric distribution). In the presence of noise (we performed 1000 realizations), the quadratic fit often gave ellipses; the orientations with the highest probability were either 30° or 120°. Therefore, we cannot discard that the elliptical shape is just a result of the sparse non-rotationally symmetric peripheral sampling. This simulation provides a potential explanation for the tilt angle (33°) observed in the contour lines, but further work seems necessary (on both experimental measurements and modeling) to understand these findings.

**Optical axis**

In the preceding paragraphs, we discussed the agreement between our experimental results and predictions of the third-order Seidel theory of aberrations. We concluded that the agreement was good if we consider a particular axis for each type of aberration, except for defocus, which provided a poor fit to the quadratic model (and of course irregular aberrations, such as trefoil, which are not included in Seidel theory). Note that this criterion to find an axis is not unique, as it changes with aberration modes. A global criterion for best optical quality axis would be the point of minimum (total) wavefront error (7.6° nasal, 7.0° inferior), but the position of this axis may be strongly biased by the possibly irregular shape of the retina, which possibly is the reason why experimental field curvature does not obey Seidel theory. On the other hand, HOA do not include oblique astigmatism, which is the main contribution in the periphery. For these reasons, we applied the quadratic model fit to the total wavefront RMS, but excluding defocus, to find the distribution of optical quality over the 2D visual field of the optical system of the eye without considering the contribution of the retinal surface. In other words, this would be the pure contribution of the optical system RMS (OS RMS). Interestingly enough, we found the best goodness of fit in this case ($R^2 = 0.9788$), and the best (global) optical quality axis is now 4.76° nasal, 1.08° inferior. This is close to the axis of astigmatism, as expected, since astigmatism is the dominant contribution in the periphery. This axis is close to the nominal value of the optical axis of the eye. In fact, this BOQ axis could be an alternative definition, based on an optical quality criterion, instead of the standard criterion of alignment of the optical surfaces of the cornea and lens.

As a further test for the consistency of the optical quality axis, we computed the geometrical center of the axes of astigmatism for the 21 field samples. Since astigmatism is dominant in the periphery, this should be a good first approximation. Briefly, the axis of astigmatism at point $(u_i, v_i)$ (with $u = \theta \cos \delta$ and $v = \theta \sin \delta$) will be a straight line passing through that point, and its slope $m_i$ would be given by the axis of astigmatism $m_i = \tan \gamma$ at point $i$th, with $\gamma = \tan^{-1}(z/(z^2/\Delta))$. The equation of that axis will be $v = v_i = m_i(u - u_i)$. The geometrical center of all these straight lines will be the point of minimum sum of distances to all of them. This is equivalent to find the point with minimum RMS difference, which is a typical
linear least squares problem. When we applied this formulation, we obtained that the coordinates of the center of minimum RMS distance between the axes of astigmatism was 6.65° nasal, 1.31° inferior retina. This is a quite reasonable agreement with the axis of best optical quality, which supports the consistency of this definition.

Physical meaning of warped ZPs

All the data analysis discussed so far was made in terms of warped Zernike modes (or warped canonical, i.e., with normalized pupil coordinates). As we said before, these warped modes have the property of keeping orthogonality within the warped pupil. In fact, the warping of coordinates (Equation 2) is a simple way of generalizing the normalization of radius for a general elliptical pupil. In addition to orthogonality, warped Zernike modes have a clear physical meaning in the case of the human eye.

Let us consider the case of pure defocus, as depicted in Figure 10. For simplicity, we display the spot diagrams computed using ZEMAX (ZEMAX Development, Bellevue, WA) on an ideal (paraxial) lens with pure defocus, on-axis (blue) and off-axis (green). The left column represents the case of a flat image plane, and the right column corresponds to a spherical image surface, more similar to the human retina. The difference between the resulting off-axis spot diagrams is patent, whereas on-axis, the spot diagrams are almost identical. In the case of spherical image surface, the spot diagram is a warped (elliptical) version of the circular pattern obtained on-axis. Since the spherical image surface is approximately normal to the beam (at least for moderate field angles), the spot diagram shows translation of its center, scaling (defocus is higher for the ideal lens), and shrinkage in the vertical direction by the cosine of the angle between the normal to the surface and the optical axis. It is worth noting that all these spot diagrams always represent pure defocus as this is an ideal aberration-free lens. Of course, this warped spot diagram comes from a warped wavefront as the coordinates of the spots are proportional to the slopes of the wavefront.

It would be straightforward to generalize this analysis to any Zernike mode. Therefore, we conclude that the warped Zernike mode has the advantages of keeping orthogonality, provide a straightforward solution to the problem of normalization of radius in elliptical pupils (providing a homogeneous canonic representation for all field angles), and what is more important, it has more physical meaning when the image surface is normal to the incident beam (what is approximately the case of the retina). If we take into account the measuring device, this formulation is especially well suited for laser ray-tracing systems that are based on imaging the spots on the retina.

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