Pupil shape as viewed along the horizontal visual field

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Changes in pupil size and shape are relevant for peripheral imagery by affecting aberrations and how much light enters and/or exits the eye. The purpose of this study is to model the pattern of pupil shape across the complete horizontal visual field and to show how the pattern is influenced by refractive error. Right eyes of 30 participants were dilated with 1% cyclopentolate, and images were captured using a modified COAS-HD aberrometer alignment camera along the horizontal visual field to ±90°. A two-lens relay system enabled fixation at targets mounted on the wall 3 m from the eye. Participants placed their heads on a rotatable chin rest, and eye rotations were kept to less than 30°. Best-fit elliptical dimensions of pupils were determined. Ratios of minimum to maximum axis diameters were plotted against visual field angle. Participants’ data were well fitted by cosine functions with maxima at (−)1° to (−)9° in the temporal visual field and widths 9% to 15% greater than predicted by the cosine of the field angle. Mean functions were 0.99 cos[(ϕ + 5.3)/1.121], R² 0.99 for the whole group and 0.99 cos[(ϕ + 6.2)/1.126], R² 0.99 for the 13 emmetropes. The function peak became less temporal and the width became smaller with increase in myopia. Off-axis pupil shape changes are well described by a cosine function that is both decentered by a few degrees and flatter by about 12% than the cosine of the viewing angle, with minor influences of refraction.

Introduction

The entrance pupil is the image of the aperture stop of an optical system viewed from the object side. For most human eyes, the pupil appears circular in shape when viewed along the line of sight (on-axis). When viewed at an angle to the line of sight, it appears elliptical with the ellipticity increasing as the angle increases. The minor axis is parallel to the meridian of the viewing angle. The size in the perpendicular direction remains approximately constant. Changes in pupil size and shape are relevant for peripheral imagery by affecting aberrations and the amount of light entering the eye (Atchison & Smith, 2000).

It might be considered that the pupil shape appears elliptical off-axis according to the cosine of the off-axis angle; that is, a pupil viewed at a peripheral angle ϕ has a ratio of minimum to maximum dimensions given by

\[
\text{ratio of ellipse minor axis to ellipse major axis} = \cos(\phi). \tag{1}
\]

This approach assumes that the aperture stop (iris) is flat and of negligible thickness and that the entrance pupil does not suffer any aberration. A ray-tracing simulation with a rotationally symmetrical model eye found that with the increasing angle the entrance pupil may be considered to move forward, tilt, and move out of a single plane. It undergoes asymmetric changes in shape, and its center no longer coincides with the on-axis center, moving in the opposite direction to that from which the pupil is viewed (Fedtke, Manns, & Ho, 2010).

Experimental studies of the size and shape of the pupil when viewed eccentrically have been restricted to the horizontal visual field. These concerned mainly the temporal side with measurements into the nasal side proceeding no further than 35° (Haines, 1969; Jay, 1961; Sloan, 1950; Spring & Stiles, 1948) apart from...
measurements of a single participant between 50° nasal and 50° temporal (Jennings & Charman, 1978) and an unpublished study measuring between 60° nasal and 100° temporal in 15 participants (Lang, 1971). Figure 1 shows results of these studies. Generally, similar results for pupil shape were found for large and small pupils (Haines, 1969; Lang, 1971; Spring & Stiles, 1948) and whether pupil size was manipulated by illumination conditions or by topical drugs (Haines, 1969). All studies found that the ratio of the minor to major pupil dimensions failed to follow Equation 1 into the temporal field with the minor diameter decreasing more slowly away from the center of the visual field than given by the equation. The failure to follow the \( \cos \phi \) relationship into the temporal field is in accordance with other findings that the eye is not optically symmetrical relative to the line of sight; for example, the best fit optical axis is located about 3°–5° temporally and 2°–3° downward in object space relative to the line of sight (Tscherning, 1900), and the pupillary axis is about 2° temporal in object space relative to the line of sight (Franceschetti & Burian, 1971; Loper, 1959). Atchison and Smith (2000) gave a fourth-order equation fitting the data from the majority of these studies in the temporal field as

\[
\frac{\text{ratio of ellipse minor axis to ellipse major axis}}{1 - 1.0947 \times 10^{-4} \phi^2 + 1.8698 \times 10^{-9} \phi^4, \quad \phi \text{ in degrees.}}
\]

The influence of refractive error on peripheral pupil shape has not been considered previously. Refractive errors might affect pupil shape through variations in axes within the eye, corneal surface curvatures and asphericities, and anterior chamber depth. Artal, Benito, and Tabernero (2006) found that angle lambda between the pupillary axis and line of sight is smaller for myopes than for hyperopes. The anterior cornea becomes more curved with increase in myopia (Atchison, 2006), while there are contrary findings about whether the cornea becomes less prolate with increasing myopia and whether the anterior chamber depth increases with increasing myopia (Atchison, 2006).

As part of a study that measured peripheral refractions of eyes along the horizontal meridian (Mathur & Atchison, 2013), we imaged the entrance pupils along the horizontal visual field between 90° temporal and 90° nasal for 30 participants of a range of refractions. The purpose of this study is to model the pattern of pupil shape across the extended horizontal visual field and to determine how this pattern is influenced by refraction. We are applying the term “horizontal visual field” loosely here as it actually extends beyond 90° on the temporal side and is limited to about 60° on the nasal side (Hefftner, 1914; Rönne, 1915; Zuckermann, 1954).

**Methods**

Participants were recruited from Queensland University of Technology and consisted of six low hyperopes (spherical equivalent +1.2 D ± 0.6 D, mean age 26 ± 5 years), 13 emmetropes (+0.0 D ± 0.3 D, 23 ± 6 years), and 11 myopes (−2.9 D ± 1.5 D, 27 ± 12 years). Emmetropes had spherical equivalents within ±0.50 D. The study complied with the tenets of the Declaration of Helsinki and was approved by the University Human Research Ethics Committee, and informed consent was obtained after explanation of the nature and possible consequences of the study. Participants had best corrected visual acuities of 6/6 or better and were screened for ocular pathology.

Right-eye images were captured using the alignment camera of a modified COAS-HD Hartmann-Shack aberrometer (Wavefront Sciences Inc., Albuquerque, NM) along the horizontal visual field in 5° steps out to
with positive angles assigned to the nasal visual field. The optical axis of the instrument was aligned with the pupil center. Left eyes were occluded. In order to maintain similar blurring for the nasal and temporal pupillary margins, the upper and lower margins were kept in focus while aligning the COAS-HD aberrometer for image capture. The right eyes were dilated with 1% cyclopentolate to avoid any effect of accommodation on pupil shape. A two-lens relay system was built on the aberrometer in order to avoid the aberrometer obstructing most of the visual field and enable fixation at the targets mounted on the wall 3 m away (Figure 2). Participants placed their heads on a rotatable chin rest, and eye rotations were kept to less than 30° (Mathur & Atchison, 2013).

Images were analyzed using a customized routine with ImageJ software (developed by Wayne Rasband, National Institutes of Health, Bethesda, MA; available at http://rsbweb.nih.gov/ij/index.html) in order to determine pupil dimensions, shape, and position relative to the limbus. The operator identified 16 points along the pupillary margin manually, and a rotated ellipse was fitted to these points using the least squares method (Morelande, Iskander, Collins, & Franklin, 2002). Similarly, an ellipse was fitted to the limbal margin. We used a manual routine because an automated routine was unable to reliably identify blurred pupillary margins at large angles. Reliable analysis of images was not possible for 31 points of 18 participants with similar proportions of participants, with missing points in each of the refraction groups. This occurred at angles ≥75° from fixation when the operator was not able to identify the nasal and/or temporal pupillary margins with confidence.

The image analysis routine gives

\[
\text{ratio} = \frac{\text{major axis of pupil ellipse}}{\text{minor axis of pupil ellipse}}
\]  

with the orientation of the major axis relative to the horizontal axis, as viewed by an observer, for an anticlockwise angle \( \theta \) from the right of the pupil of between 0° and 180° (Figure 3).

A ratio of the horizontal to vertical dimensions of the ellipse \( A \) was given by

\[
A = \frac{1}{\text{ratio}}, \quad 45° < \theta < 135°,
\]

\[
A = \text{ratio}, \quad \theta \leq 45° \quad \text{and} \quad \theta \geq 135°.
\]
This is an approximate horizontal/vertical ratio as we take the “horizontal” dimension as the ellipse axis within 45° of the horizontal meridian. We determined $B$, the horizontal-vertical component of the pupil ellipticity, and $C$, the oblique component of the pupil ellipticity, as

$$B = (1 - A) \cos[2(\theta - 90)] \quad (5)$$

$$C = (1 - A) \sin[2(\theta - 90)] \quad (6)$$

$B$ and $C$ give information on the orthogonal and oblique components of the pupil ellipticity. If the minor axis is always horizontal, $B$ will be $(1 - A)$, and $C$ will be 0. Three images were taken for each position, with $A$, $B$, and $C$ components averaged across them.

Cosine fits were made to the average $A$ values as

$$y(\phi) = D \cos \left[ \frac{\phi - \beta}{E} \right], \quad (7)$$

where $D$ is the amplitude of the fit, $\phi$ is visual field angle (negative/positive for temporal/nasal visual field), $\beta$ is the peak of the fit relative to the center of the visual field, and $180E$ is half the period of the fit in degrees. These parameters are shown in Figure 4.

**Results**

Figure 5 shows a set of images for one participant across the visual field. The horizontal pupil dimension reduces more quickly for the nasal field than for the temporal visual field.

Figure 4 shows $A$, $B$, and $C$ for one participant. The main features are (a) the excellent fit of Equation 7 to the results across the $\pm 90^\circ$ field with an adjusted $R^2$ of 1.00; (b) the function is 1.15 times wider than predicted by the cosine of the visual field angle; (c) the peak of the function is offset to the temporal field side by $(-)7.6^\circ$; and (d) the oblique component of pupil ellipticity $C$ follows a linear relationship with the field angle.

Across participants, the ranges of parameters were amplitude $D$ 0.94 to 1.04, offset $\beta$ $-8.5^\circ$ to $-0.9^\circ$, and $E$ 1.09 to 1.15 with adjusted $R^2$ values varying from 0.98 to 1.00. Using $t$ tests with the regression data, $D$ was found to be significantly different from 1 in all but five cases, $\beta$ was significantly less than 0 in all but one case, and $E$ was significantly >1 in all cases.

One approach to combining the data across participants is to average the $D$, $\beta$, and $E$ values separately, and a second approach is to combine the data of all participants and fit using Equation 7 as shown in Figure 6. The disadvantage with the first approach is that it ignores possible interdependence of the variables while the second approach treats the individual points for each participant as independent and does not account for missing points. The first approach gives

$$y(\phi) = 0.933 \pm 0.005 \times \cos \left( \frac{\phi + 5.3 \pm 0.3}{1.120 \pm 0.003} \right) \quad (8)$$

and the second approach gives

$$y(\phi) = 0.992 \pm 0.002 \times \cos \left( \frac{\phi + 5.3 \pm 0.1}{1.121 \pm 0.002} \right), \quad (9)$$

where the numbers given in italics are standard errors. The two approaches give similar parameters, including amplitudes very close to unity, and $t$ tests do not show any significant differences.
When the oblique component of the pupil ellipticity $C$ is treated in a similar fashion, the first approach gives

\[ C = +0.00072 \pm 0.00006 \times \phi - 0.0111 \pm 0.0019 \] (10)

and the second approach gives

\[ C = +0.00072 \pm 0.00002 \times \phi - 0.0120 \pm 0.0008. \] (11)

The fit for the second approach is shown in Figure 6. Again, the two approaches give similar parameters. The slopes and vertical axis intercepts are significantly different from zero. The positive slope means that $\theta$ in Equations 5 and 6 and Figure 3 is less than 90° for negative (temporal) visual field angles and greater than 90° for positive (nasal) visual field angles. As viewed by an observer, the top of the pupil is angled slightly away from him or her.

It is expected that the vertical dimension of the pupil should change little with visual field position. In agreement with previous studies, this is confirmed in Figure 7. It is only beyond 80° nasal field that the vertical dimension becomes unreliable.

The position of symmetry $b$ and the period $E$, but not the amplitude $D$, are significantly influenced by refraction. The myopic group has significantly more negative $b$ than the emmetropic group (mean difference 2.3°, $p = 0.01$) and significantly lower $E$ than the emmetropic (0.019, $p = 0.003$) and hyperopic (0.022, $p = 0.029$) groups. There are moderate trends for $b$ and $E$ with $b$ becoming more negative as refraction becomes less negative/more positive (Figure 8) and $E$ becoming greater as refraction becomes less negative (Figure 9).

The fit of the 13 emmetropic patients according to our second approach gives

\[ y(\phi) = 0.987 \pm 0.002 \times \cos \left( \frac{\phi + 6.2 \pm 0.1}{1.126 \pm 0.002} \right). \] (12)

The fit has not been included in Figure 6 because it is almost indistinguishable from the fit for all participants.
Discussion

Figure 10 shows the results of some previous studies and the mean fit from this study. This study supports previous studies (Haines, 1969; Jay, 1961; Lang, 1971; Sloan, 1950; Spring & Stiles, 1948) in finding that the shape of the pupil does not change as quickly when it is viewed at increasing angles along the horizontal visual field as predicted by the cosine of the viewing angle. We found that the rate of change of shape is about 90% of that predicted by the cosine of the viewing angle. This is a slightly higher rate than that obtained by a theoretical study (Fedtke et al., 2010). Our study shows that Equation 2 for the temporal field produced by Atchison and Smith (2000), as a summary of previous studies mainly limited to the temporal side of the field, underestimates the rate of change of pupil shape in the nasal field (Figure 10). To the contrary, the assumption that the pupil shape changes according to the cosine of the visual field, while inappropriate for the temporal field, holds well for the nasal field.

Pupil shape is described well by a cosine function that has a peak a few degrees into the temporal field. This is consistent with other findings concerning symmetry in relation to the line of sight (or the visual axis). Tscherning (1900) reported over a hundred years ago that the best fit optical axis is about 3°–5° temporally relative to the line of sight. The pupillary axis (the line passing through the pupil center and the center of curvature of the cornea) is a few degrees on the temporal side of the line of sight in object space (Artal et al., 2006; Franceschetti & Burian, 1971; Loper, 1959), and the turning (stationary) point of peripheral astigmatism is a few degrees into the temporal visual field (Atchison, Pritchard, & Schmid, 2006; Dunne, Misson, White, & Barnes, 1993; Lotmar & Lotmar, 1974). We have data on the last of these for our participants. From taking quadratic fits of data across the central ±35° of the horizontal visual field, we obtained turning points of the horizontal/vertical astigmatism. The mean and standard error are −4.9 ± 0.9° as compared with those for the pupillary peaks of −5.3 ± 0.3°; the means are similar.

The pupil shape symmetry position moves closer to the line of sight as myopia decreases (Figure 8). This is consistent with findings for astigmatism and the finding of Artal et al. (2006) that angle \( \lambda \) is smaller for myopes than for hyperopes. The rate at which the pupil changes shape increases as the refraction moves in the myopic direction (Figure 9).

The finding that there is an oblique component to the pupil shape, that is, the major axis of the pupil ellipse does not remain vertical but rotates away from the observer at the top, is consistent with the pupil having its orientation downward relative to the line of sight (Figure 11), similar to the best fitting optical axis relative to the visual axis (Tscherning, 1900).

There are some limitations to this study. First, pupils were dilated with the anticholinergic drug cyclopentolate, and so the effects of different drug states and light levels were not explored. In his on-axis study, Wyatt (1995) found a tendency for the major axis of the natural pupil to change from vertical in the dark to
horizontal in the light. This is supported to a small extent by Lang (1971): Fitting his data to Equation 7 gives an increase in amplitude $D$ from 0.976 in the dark to 0.992 at a high light level. However, Haines’ results (1969) were in the opposite direction: 0.99 in the dark to 0.94 at a high light level. Haines’ results for cyclopleged and naturally large pupils were similar with no statistical difference in the amplitudes ($D$).

A second limitation is that we fitted ellipses to pupils, which may be an oversimplification, particular at large visual field angles. Third, only the horizontal visual field was considered.

Our interest in pupil shape arose because of its influence on peripheral aberrations and image quality. Many researchers in peripheral optics have ignored the elliptical pupil shape while we have assumed that it changes according to the cosine of visual field angle (Charman, Mathur, Scott, Hartwig, & Atchison, 2012; Mathur, Atchison, & Scott, 2008). Using the average fit of Equation 7 shows that the cosine of the angle assumption (Equation 1) will give $<$10% incorrect estimates of size between 40° temporal and +70° nasal while using the on-axis pupil size will give overestimates of $<$10% within a smaller range of 30° temporal to +20° nasal. This suggests that the cosine assumption is reasonable within a large range of visual field angles.

The findings in this study should be regarded when determining retinal illuminance, aberrations, and image quality in the peripheral visual field. As an example, Equation 7 could be used for providing pupil dimensions for point spread function and optical transfer functions.

**Conclusion**

Off-axis pupil shape is well described by a cosine function that is both decentered by a few degrees and flatter by about 12% than the cosine of the viewing angle. There are minor influences of refraction.

**Keywords:** peripheral vision, pupil shape

**Acknowledgments**

This work was supported by ARC Discovery grant DP110102018 and ARC Linkage Grant LP100100575 and by an Institute of Health and Biomedical Innovation Early Career Research Grant to Ankit Mathur. We thank the Research Division of Carl Zeiss Vision for support.

Commercial relationships: none.
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