Perception of multi-stable dot lattices in the visual periphery: An effect of internal positional noise

Endel Põder

Institute of Psychology, University of Tartu, Tartu, Estonia

Dot lattices are very simple multi-stable images where the dots can be perceived as being grouped in different ways. The probabilities of grouping along different orientations as dependent on inter-dot distances along these orientations can be predicted by a simple quantitative model. L. Bleumers, P. De Graef, K. Verfaillie, and J. Wagemans (2008) found that for peripheral presentation, this model should be combined with random guesses on a proportion of trials. The present study shows that the probability of random responses decreases with decreasing ambiguity of lattices and is different for bi-stable and tri-stable lattices. With central presentation, similar effects can be produced by adding positional noise to the dots. The results suggest that different levels of internal positional noise might explain the differences between peripheral and central proximity grouping.

Keywords: peripheral vision, grouping, proximity, orientation, positional noise


Introduction

Lattices of regularly spaced dots are well-known examples that demonstrate the Gestalt principles of grouping by proximity and similarity. They are also very simple multi-stable (ambiguous) images. For example, the dots in Figure 1A can be perceived equally often as grouped along rows or columns; those in Figure 1B are much more likely perceived as grouped along rows.

Oyama (1961) made the first attempt to quantify the relationship between distances between dots and perceptual dominance of vertical vs. horizontal grouping. He found that the ratio of durations of seeing one or another grouping is a power function of the ratio of distances between dots along the rows and the columns.

Kubovy and Wagemans (1995) made several improvements to the experimental methods (random absolute orientation, circular aperture, short exposure) and also used different types of lattices besides the simple rectangular ones (see Figure 1C). These authors suggested a new model to predict the probabilities of perception of the different possible grouping orientations (Kubovy, Holcombe, & Wagemans, 1998). The model assumes that the probability of perceiving a given grouping orientation follows an exponential decay function of the relative distance between the nearest dots in a given orientation. More exactly, the ratio of two probabilities is given by

\[
p(b) = \frac{\exp \left( -k \left( \frac{b}{a} - 1 \right) \right)}{p(a)},
\]

where \(a\) and \(b\) are inter-dot distances and \(k\) is the free distance sensitivity parameter (attraction constant).

This model is also known as the logit or logistic regression, which is widely used to predict probabilities of choosing between different alternatives in many fields of social sciences. Kubovy et al. (1998) refer to this model as the Pure Distance Law (PDL). Several studies have demonstrated that it usually fits the data on proximity grouping within dot lattices well (e.g., Claessens & Wagemans, 2005, 2008; Kubovy & van den Berg, 2008). Still, there are other mathematical equations that can be equally good and may be preferred for some theoretical reasons (e.g., Claessens & Wagemans, 2008; Geisler & Super, 2000).

Kubovy et al. (1998) and Kubovy and Wagemans (1995) presented their stimuli at the fixation point. In a recent study, Bleumers, De Graef, Verfaillie, and Wagemans (2008) tested peripheral locations with similar stimuli. They found that the simple PDL did not fit the peripheral grouping data. However, the fit was much better when the PDL was combined with an assumption of occasional attention lapses. It was assumed that on some proportion of trials, observers could not attend the stimulus and responded with random guesses. However, the reason for these lapses remains unclear. It is possible that they are not related to visual processing in the periphery at all but were caused by some properties of the experimental procedure (e.g., requirement to maintain fixation and at the same time allocate attention peripherally). Furthermore, the estimated parameters of the PDL model showed that observers were more sensitive to changes of inter-dot distances in the visual periphery as compared with a central location. This seems curious, though not impossible. Anyway, a combination of two effects (lapses of attention and higher sensitivity to inter-dot distances) does not seem parsimonious enough, and one may hope to find a simpler account. In addition, Bleumers et al. (2008) used...
the simplest rectangular lattices only, and their results are not necessarily valid for other types of lattices.

The purpose of the present study was to understand the difference between the fovea and periphery better in regard to proximity grouping within dot lattices. I used the dot lattice stimuli, similar to Bleumers et al.’s (2008) and Kubovy et al.’s (1998) studies. In order to avoid problems with the control of fixation, a very brief presentation of stimuli, unpredictably either left or right from the fixation, was used. Trying to discriminate the stimulus-dependent and random responses better, I used a response panel with more alternatives that covered the full range of orientations uniformly. Besides the rectangular lattices used by Bleumers et al. (2008), I included another distinct type of lattice, which can be loosely named “triangular.” In the exploratory Experiment 1, both central and peripheral presentations were used, and the size of the stimuli was varied over a large range. In Experiment 2, a larger number of trials were run with peripheral presentation and fixed stimulus size.

Methods

Experiment 1

The stimuli were presented on a 15-inch CRT monitor with a resolution of 1024 × 768 pixels. Examples of stimuli are shown in Figure 2. The dots were black on a white background. The lattice had a circular outline shape with a black border. Three different sizes of lattices were used, with a radius of 25, 50, and 100 pixels. The radii of the dots were 1, 2, and 4 pixels, and the smallest distance between the dots was 8, 16, and 32 pixels, respectively. With a viewing distance of about 60 cm, the radius of the lattice was 0.8, 1.6, or 3.2 deg.

I used two types of symmetrical lattices: rectangular, with two dominant grouping orientations (square and rectangular lattices, according to Kubovy, 1994) and triangular, with three dominant orientations (hexagonal and centered rectangular, according to Kubovy, 1994; see Figure 2). The ratio of distances between the dots in the different orientations $b/a$ (a measure of dominance of the orientation $a$) was varied from 1 to 1.5 in 0.125 steps (see Figure 2). The absolute orientation of the lattices was random.

The lattices were presented either centrally (at the fixation point) or at 9-deg eccentricity (measured from the center of the lattice), randomly either left or right. The exposure duration was 60 ms. After the exposure, observers had to indicate the orientation of the perceived grouping by clicking an icon with closest to the perceived orientation in a response panel (see Figure 2).

Two observers took part in this experiment (EP is the author and CM was naive with respect to the purpose of the study). They both ran 2400 trials in total. The size of the lattice and eccentricity were held constant within blocks of 50 trials. The type of lattice and the $b/a$ ratio were varied within blocks.

Figure 1. Examples of multi-stable dot lattices. Rectangular lattices with (A) equal and (B) different inter-dot distances along the rows and columns. (C) A lattice with three dominant orientations, with the three inter-dot distances indicated.

Figure 2. Examples of stimuli used in this study. (A, C) Rectangular lattices and (B, D) triangular lattices, with $b/a$ ratios of (A, B) 1 and (C, D) 1.5. (E) Response panel with 8 orientation alternatives.
Experiment 2

In this experiment, only the peripheral presentation (eccentricity 9 deg) was used. The radius of the lattice was fixed at 50 pixels (1.6 deg), with a minimal inter-dot distance of 16 pixels and dot radius of 2 pixels. Otherwise, the methods were identical to Experiment 1. Two observers (the author and another naive observer) participated in this experiment. They both ran 1500 trials.

Data analysis and modeling

The data from the present experiments are proportions of responses across 8 orientation categories. For the analysis, the categories were positioned relative to the a orientation of a lattice in each trial (the center of the category comprising a orientation was assigned orientation zero). The data are pooled across absolute orientations. For better reliability, I also averaged over negative and positive (clockwise and anticlockwise) orientations relative to the a orientation. Thus, a data set for five b/a ratios and two lattice types has 40 degrees of freedom.

The analysis and modeling are based on a few simple and natural assumptions. When a multi-stable dot lattice is presented, an observer perceives a grouping in one of the several possible orientations. The perceived orientation does not match an orientation present in the lattice exactly but is perturbed by some amount of internal noise. I assume that this noise is Gaussian. The observer chooses the response alternative that is closest to the perceived orientation. When an observer fails to see any grouping, he/she selects a response alternative randomly. Thus, the distribution of responses is a mixture of several Gaussian distributions corresponding to the dominant orientations within the lattice and the uniform distribution of random guesses (Figure 3A). Mathematically, response probability is given by

\[
p(\theta) = \sum_v p_v \phi(\theta, \theta_v, \sigma) + \frac{p_r}{180},
\]

where \(p_v\) is proportion of responses based on dominant grouping orientation \(v \in \{a, b, c, \ldots\}\), \(\phi(\theta, \theta_v, \sigma)\) is Gaussian distribution with mean \(\theta_v\) and standard deviation (SD) \(\sigma\), \(p_r\) is proportion of random guesses (distributed uniformly across 180 deg—the full range of orientations). When using PDL, \(p_v = p(v)(1 - p_r)\), where \(p(v)\) is proportion of responses \(v\) predicted by that model.

An additional step is necessary to account for an interaction of the fixed set of response categories with random orientations of the presented lattices. When measured relative to the center of response category zero, the same dominant orientations within a lattice do not coincide across different trials but form 22.5-deg bands with a uniform probability density. Therefore, the response distributions corresponding to the dominant orientations of a lattice are not Gaussians any more but the convolutions of the Gaussians with the 22.5-deg-wide rectangular distribution. The effect of this step is shown in Figure 3B.

The final continuous distribution of predicted responses was broken down into 8 response categories. Because the orientation is a circular variable (with a period of 180 deg), all these calculations were done on a circular axis. (Theoretically, wrapped Gaussian distributions should be used with this kind of axis. However, with SD small relative to the period (as in this study), there is practically no need to wrap a distribution over more than a single cycle; only periodic boundary conditions must be taken into account.)

Within this general framework, it is possible to estimate meaningful parameters from the data or to fit different theoretical models. I used MS Excel Solver to search for optimal values of the parameters. The fit was measured by the following likelihood ratio statistic:

\[
G = 2 \sum O_{ij} \ln \left( \frac{O_{ij}}{E_{ij}} \right),
\]

where \(O_{ij}\) and \(E_{ij}\) are observed and expected counts, respectively.
where $O_{ij}$ is observed and $E_{ij}$ is the expected frequency in cell $i,j$. For the exploratory Experiment 1, I found the proportions ($p_v$) of responses based on each of the dominant orientations of a lattice, the proportion of random guesses ($p_r$), and SD of orientation errors ($\sigma$) that accounted for the observed response distributions best. Because of the small number of trials per condition, I used only the 2 and 3 most dominant orientations for rectangular and triangular lattices, respectively.

For the data from Experiment 2, three models were fitted. Model 1 is the simple PDL (Kubovy et al., 1998), which predicts the proportions of responses ($p_v$) along four dominant orientations, combined with Gaussian orientation errors and implemented within the present framework with 8 response alternatives. The model has two free parameters: distance sensitivity parameter of PDL ($k$) and SD of orientation errors ($\sigma$).

Model 2 is the PDL combined with random guessing in a proportion of trials (Bleumers et al., 2008) and also includes Gaussian orientation errors. It has three parameters: distance sensitivity, SD of orientation errors, and proportion of random guessing ($p_r$).

Model 3 is identical to Model 2, except for the separate proportions of random guessing for different lattice types and $b/a$ ratios. The model has 12 parameters in total: distance sensitivity, SD of orientation errors, and 10 proportions of random guessing.

For the comparison of models with different numbers of free parameters, I used $G$-tests based on chi-square distribution and Akaike’s information criterion with small sample correction $AIC_C = -2 \ln(L) + \frac{2k_n}{n-k_0-1}$, where $L$ is the maximum of the likelihood function of the model, $k$ is number of free parameters, and $n$ is number of independent data points (Burnham & Anderson, 2002).

## Results

### Experiment 1

The main raw data were the distributions of responses relative to the main orientation of a lattice. Examples are given in Figure 4. Already the raw data show that with small lattices and peripheral presentation, the distributions of responses become nearly uniform, indicating a prevalence of random guesses. In addition, the occurrence of random responses does not seem to be independent of stimuli but is more prevalent with a small $b/a$ ratio.

In order to see how eccentricity, stimulus size, lattice type, and $b/a$ ratio affect the important aspects of the response distributions, I used simple atheoretical modeling. From the observed response distributions, I estimated the: (1) proportions of responses based on supposedly salient orientations within a lattice ($a, b, c$), (2) SD of a perceived orientation ($\sigma$), and (3) proportion of random responses ($p_r$). SD of orientation errors was assumed to be invariant across the $b/a$ ratios; the proportions of response categories were estimated separately for each condition.

Because of a small number of trials per condition, the estimated proportions are rather noisy. Still, the results exhibit several consistent and interesting regularities. The main results are given in Figure 5 (in these graphs, the results are averaged over three lattice sizes). These data show clearly that the proportion of random guessing is much larger for the peripheral presentation and decreases dramatically with $b/a$ ratio. There was also a decrease of random responses with increasing stimulus size (not shown in the figure).

Interestingly, there are much more random responses with triangular (tri-stable) lattices, indicating that performance with peripheral presentation is not determined just by the smallest inter-dot distance and $b/a$ ratio but depends on lattice type as well. SD of orientation errors was slightly larger for peripheral presentation (CM, 12.4 deg; EP, 10.9 deg) as compared with central presentation (CM, 10.7 deg; EP, 7.9 deg). There were no systematic differences in this parameter across different sizes or lattice types.

![Figure 4](https://jov.arvojournals.org/pdfaccess.ashx?url=/data/journals/jov/933481/) Results of Experiment 1. Examples of response distributions relative to dominant grouping orientation for different $b/a$ ratios.
The purpose of this experiment was to get more reliable data on grouping in the visual periphery and test several quantitative models. Therefore, a larger number of trials were run with the peripheral presentation only. In addition, the size of lattices was not varied in this experiment.

The results replicated all the important aspects found in Experiment 1 with peripheral presentation. I fit three models to these data: (1) Pure Distance Law (Kubovy et al., 1998), (2) PDL combined with a fixed proportion of

**Figure 5.** Results of Experiment 1. Estimated proportions of random responses (r) and responses corresponding to the dominant orientations (a, b, and c) of a lattice as dependent on b/a ratio, lattice type, and eccentricity.

**Experiment 2**

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The results replicated all the important aspects found in Experiment 1 with peripheral presentation. I fit three models to these data: (1) Pure Distance Law (Kubovy et al., 1998), (2) PDL combined with a fixed proportion of
random guessing (Bleumers et al., 2008), and (3) PDL with a stimulus-dependent proportion of random responses. In order to calculate the predicted response distributions, these models were combined with a Gaussian distribution of orientation errors (SD was adjusted together with other free parameters). The fits are given in Table 1, and the optimal values of parameters are in Table 2.

For both observers, the PDL could not fit the data, and a fixed proportion of random guessing did not help, either (see Table 1). Only allowing the proportion of random responses to depend on the \( b/a \) ratio produced an acceptable fit (no statistically significant difference between the model and data). Still, the tests reported in Table 1 do not imply that the third model fits the data significantly better than two others. As the models are nested, the differences in fit can be directly tested using the differences in the values of G-statistics. These direct pairwise tests too showed highly significant \( p < 10^{-4} \) superiority of the third model over two others for both observers. In addition, I calculated an information criterion (AICc) that takes into account both fit and number of parameters (the smallest value corresponds to the best model). This criterion (Table 3) also favors Model 3.

The estimated proportions of random responses are shown in Figure 6. Although the absolute values are different, the curves are qualitatively similar across the observers and similar to the results from Experiment 1. Again, the striking difference between the two lattice types was observed. For the same proportion of random responses, the \( b/a \) ratio must be considerably larger with triangular as compared with rectangular lattices.

Table 1. Fits of the three models to the data from Experiment 2. Values of G-statistic and statistical significance of differences between the model and the data.

<table>
<thead>
<tr>
<th>Observer</th>
<th>PDL</th>
<th>PDL + fixed proportion random</th>
<th>PDL + varied proportion random</th>
</tr>
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<tbody>
<tr>
<td>EP</td>
<td>66.4 ( (p &lt; 0.001) )</td>
<td>66.4 ( (p &lt; 0.001) )</td>
<td>30.4 (n.s.)</td>
</tr>
<tr>
<td>DI</td>
<td>114.2 ( (p &lt; 0.001) )</td>
<td>112.4 ( (p &lt; 0.001) )</td>
<td>41.0 (n.s.)</td>
</tr>
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</table>

Table 2. Parameters of the three models fit to the data from Experiment 2.

Discussion, supplementary experiment, and simulation model

The two experiments demonstrated that consistent with Bleumers et al. (2008), the grouping task with peripherally presented dot lattices produces random guesses in a proportion of trials. However, the present results cannot be explained by stimulus-independent lapses of attention. Both experiments show that the probability of random responses is strongly affected by the \( b/a \) ratio and type of lattice. It seems that with peripheral presentation, grouping performance is limited by some kind of noise that masks relatively weak and ambiguous orientation signals while the \( b/a \) ratio is close to 1. With increasing \( b/a \) ratios, not only the relative but also absolute strength of a dominant orientation signal probably increases, making the grouping along that orientation visible within the noise. The characteristic differences between the two lattice types suggest that a simple positional noise might play a role in these results. A close look at the lattices used in this study reveals that regardless of the same minimal inter-dot distance and the same \( b/a \) ratio, the triangular lattices have smaller average distances between the dots. Consequently, adding the same 2D noise to the positions of the dots must distort this lattice more severely. In addition, several studies have reported spatial uncertainty as an important property of peripheral vision and measured internal positional noise at different eccentricities (e.g., Hess & Dakin, 1999; Hess & Hayes, 1994; Levi, Klein, & Aitsebaomo, 1985).

If the random responses with peripheral presentation are caused by internal positional noise, then it should be possible to produce similar results by adding external noise to centrally presented stimuli. I tested this prediction in a supplementary experiment. The methods were identical to those of Experiment 2, except the lattice was presented at the fixation point and Gaussian noise (SD 3 pixels) was added to the \( x \) and \( y \) coordinates of each dot. Examples of stimuli are given in Figure 7. Two observers participated in the experiment (600 trials). Figure 7 depicts the proportions of random responses, estimated in the same way as in Experiment 2. The pattern of results is really similar to those of Experiment 2. Both observers exhibit a decreasing proportion of random responses with increasing \( b/a \) ratios and a higher proportion of random
responses with triangular lattices. Thus, the results show that adding positional noise to the dots of a centrally presented lattice produces the same effects as presenting a noiseless lattice in the visual periphery. This is strong support for the hypothesis that internal positional noise in the visual periphery might be the main cause of the central–peripheral differences in proximity grouping.

Finally, I attempted to build a simple computational model (Figure 8) that could take the images of dot lattices as input and produce response distributions similar to those found in the experiments. I used a set of spatial filters tuned to 16 different orientations (evenly distributed, step 11.25 deg) and 9 wavelengths (from 4 to 12 pixels). The filters had symmetrical (cosine phase) Gabor profiles with circular Gaussian window (standard deviation equal to the wavelength). Spatially, the filters were positioned at the center of a lattice. The size of images was 63 × 63 pixels, and the minimal distance between dots was 8 pixels. Pixel values were 1 for the dots and 0 for background. The model calculated the filter responses (weighted sum, dot product) to a given lattice and added independent Gaussian noise (standard deviation 2) to the results. (Without that noise, the model would always choose the same response when the same noise-free lattice is presented.) The filter with maximum response was selected, and its orientation was used as the model’s response in a given trial. I generated simulation results for the lattices of different types, with different $b/a$ ratios, with and without positional noise (300 simulated trials per condition). For a better comparability with the experimental data, I convolved the response distributions with a typical distribution of human orientation errors ($SD$ 10 deg) and presented the results in 8 orientation categories.

The results (see Figure 8) show that this simple model reproduces virtually all qualitative regularities of the human data. The simulated data without positional noise are well in accord with PDL ($p > 0.9$). With positional noise, the estimated proportions of random responses follow the same pattern as observed in the experiments with human observers exposed to the similar noisy stimuli, or when noiseless stimuli are presented in the visual periphery. Thus, the simulation provides an additional support to the idea that positional noise can produce the results that are observed in experiments with peripheral presentation of dot lattices.

The results of the present study look a bit different from Bleumers et al.’s (2008) conclusions. These authors claimed that stimulus-independent random guesses could explain their results from peripheral presentation. There

<table>
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<tr>
<td>EP</td>
<td>3771</td>
<td>3776</td>
<td>3766</td>
</tr>
<tr>
<td>DI</td>
<td>3767</td>
<td>3770</td>
<td>3725</td>
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Table 3. Akaike’s information criterion ($AIC_C$) for the three models fit to the data from Experiment 2.
are several reasons why it could be difficult to observe the effect of $b/a$ ratio on the probability of random responses in that study. They varied $b/a$ ratio over a relatively small range (from 1 to 1.26). Their lattices were somewhat larger (relative to eccentricity) and exposure duration was longer, which should reduce the total amount of random responses. They did not use tri-stable lattices that produced the largest proportion of random responses in the present study. In addition, note that only using response alternatives far from dominant grouping orientations (as in the present study) allows a robust estimation of the probability of random responses. However, it is not impossible that their procedure really produced stimulus-independent lapses of attention on some proportion of trials. (I reanalyzed their results and found a significant decrease of random responses with increasing $b/a$ ratios at Figure 8. Simulation of proximity grouping within dot lattices. (A) A dot lattice superimposed with a spatial filter (in total, 144 filters with 16 orientations and 9 sizes were used). Simulated response distributions (B) without and (C) with positional noise. (D) Estimated proportion of “random guesses” for two types of lattices with positional noise.
15-deg eccentricity in their Experiment 1 but not in Experiment 2.)

In this study, I followed Bleumers et al.’s (2008) idea that deviation from PDL in the visual periphery can be explained by random guessing in a proportion of trials. An alternative possibility of increased orientation errors looks less likely. Supposedly, the orientation errors are introduced at relatively late stages after the grouping and seem to be independent of \( b/a \) ratio and lattice type (Kubovy et al., 1998). In addition, I have found that using variable SD of orientation errors could not fit the present data as well as variable proportions of random responses.

Although I used the PDL (Kubovy et al., 1998) as a kind of baseline model, the results of the present study do not favor this or any other specific function used to quantify proximity grouping. However, the present findings point at some limitations of Kubovy et al.’s (1998) assumption that relative inter-dot distances along different orientations solely determine the probabilities of perception of differently oriented groupings while the angles between orientations (or lattice types) do not matter. This seems to be correct for central noiseless presentation but not for peripheral presentation or for lattices with positional noise. The simulations run in the present study indicate that it is possible to build a biologically plausible computational model of proximity grouping that reproduces the regularities reported in previous studies (e.g., Kubovy et al., 1998; Kubovy & Wagemans, 1995) and also accounts for the effects of (internal or external) positional noise.

**Conclusions**

This study shows that differences between the fovea and periphery in proximity grouping within dot lattices cannot be explained by stimulus-independent lapses of attention and suggests that the main differences are caused by increased positional noise in the visual periphery. Both peripheral and central proximity grouping data can be reproduced by a relatively simple computational model based on classical spatial filters tuned to various orientations and spatial frequencies.

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Corresponding author: Endel Pöder.
Email: endel.poder@ut.ee.
Address: Institute of Psychology, University of Tartu, Tiigi Street 78, Tartu 50410, Estonia.

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