The change of spherical aberration during accommodation and its effect on the accommodation response

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Theoretical and ray-tracing calculations on an accommodative eye model based on published anatomical data, together with wave-front experimental results on 15 eyes, are computed to study the change of spherical aberration during accommodation and its influence on the accommodation response. The three methodologies show that primary spherical aberration should decrease during accommodation, while secondary spherical aberration should increase. The hyperbolic shape of the lens’ surfaces is the main factor responsible for the change of those aberrations during accommodation. Assuming that the eye accommodated to optimize image quality by minimizing the RMS of the wave front, it is shown that primary spherical aberration decreases the accommodation response, while secondary spherical aberration slightly increases it. The total effect of the spherical aberration is a reduction of around 1/7 D per diopter of stimulus approximation, although that value depends on the pupil size and its reduction during accommodation. The apparent accommodation error (lead and lag), typically present in the accommodation/response curve, could then be explained as a consequence of the strategy used by the visual system, and the apparatus of measurement, to select the best image plane that can be affected by the change of the spherical aberration during accommodation.

Keywords: accommodation, aberrations, primary spherical aberration, secondary spherical aberration, accommodation response


Introduction

The fundamentals of the mechanism of human accommodation were already studied in the 17th century by Thomas Young, who showed that the eye is able to generate a change in focus mainly by changing the curvature of the front surface of the lens (Young, 1801). Young’s findings have been totally corroborated by many in vivo (Jones, Atchison, Meder, & Pope, 2005; Koretz, Bertasso, Neider, True-Gabert, Kaufman, 1987; Neider, Crawford, Kaufman, & Bito, 1990; Strenk et al., 1999) and in vitro (Glasser & Campbell, 1998; Manns et al., 2007) measurements using different techniques that show, with great detail, the changes of the lens during accommodation. Wave-front technology has been mainly used during the last 15 years in the eye to get objective in vivo measurements of how the power of the eye changes when the light passes through different parts of the pupil (Howland & Howland, 1977; Liang, Grimm, Goelz, & Bille, 1994; Sminov, 1961). This technology has also been applied to the accommodated eye, showing a curious effect: the power of the eye, which is usually larger in the periphery of the pupil than at its center, varies during accommodation in such a way that in the accommodated eye it is larger in the center than in the periphery of the pupil (Atchison, Collins, Wildsoet, Christensen, & Waterworth, 1995; Buehren & Collins, 2006; Cheng et al., 2004; He, Burns, & Marcos, 2000; Ivanoff, 1947; Kooman, Tousey, & Scolnik, 1949; López-Gil et al., 2008; Ninomiya et al., 2002; Plainis, Ginis, & Pallikaris, 2005; Radhakrishnan & Charman, 2007; Tscherning, 1900). That power distribution is related to the radial symmetrical high-order terms of the ocular wave front, usually known as spherical aberration (SA).

For clarity, it is important to point out that there are several mathematical expressions for SA depending on the polynomial expansion used and the degree of the expansion. For simplicity, in this article, we will use the expansion of the wave front in terms of Seidel and Schwarschild aberrations (Mahajan, 1991):

$$W(r) = A_d \left( \frac{r}{r_0} \right)^2 + A_s \left( \frac{r}{r_0} \right)^4 + B_s \left( \frac{r}{r_0} \right)^6,$$  \hspace{1cm} (1)

where $r$ is the radial coordinate of the entrance pupil, and $r_0$ is the actual entrance pupil radius. $A_d$ corresponds to the coefficient of the defocus, while $A_s$ and $B_s$ correspond to primary (SA4) and secondary (SA6) spherical aberrations, respectively.
Actually, Zernike polynomial expansion is also being used broadly, for which the wave front in Equation 1 would be written as (Thibos, Applegate, Schwiegerling, Webb, & VSIA Standards Taskforce Members, 2002):

\[
W(\rho) = a_2^0 \sqrt{3} (2 \rho^2 - 1) + a_4^0 \sqrt{5} (6 \rho^4 - 6 \rho^2 + 1) \\
+ a_6^0 \sqrt{7} (20 \rho^6 - 30 \rho^4 + 12 \rho^2 - 1),
\]

where \( \rho = \frac{s}{r} \), \( a_2^0 \) is the defocus Zernike term, and \( a_4^0 \) and \( a_6^0 \) are the fourth- and sixth-order Zernike spherical aberration terms, respectively. Equations 1 and 2 show that Seidel defocus \( (A_\omega) \) and Zernike defocus \( (a_4^0) \) coefficients are directly proportional, as well as primary Seidel SA and fourth-order Zernike \( (a_4^0) \) SA, and secondary Schwarschild \( (B_j) \) and sixth-order Zernike SA \( (a_6^0) \). In particular, for a Zernike expansion up to the sixth order, the relationship between Seidel–Schwarschild and Zernike SA coefficients can be calculated from Equations 1 and 2, giving:

\[
a_4^0 = \frac{A_s}{6 \sqrt{5}} + \frac{B_s}{4 \sqrt{5}},
\]

\[
a_6^0 = \frac{B_s}{20 \sqrt{7}}.
\]

There is a complete agreement among researchers about the decrease (change in the negative direction) in \( A_\omega \) (or \( a_4^0 \)) during accommodation; in fact, this effect was already known by researchers in the first half of the last century. As a matter of fact, Young (1801), and later Tscherning (1900), using what can be assumed to be the first subjective aberroscope, showed that the eye had positive spherical aberration \( (A_\omega) \), whose magnitude decreases during accommodation. Similar results using other techniques were obtained by Ames and Proctor (1921), Ivanoff (1947), Jenkins (1963), Kooman et al. (1949), and Sminov (1961). In the last two decades, several other studies, most of which made use of aberrometric techniques, showed similar results both after in vivo measurements (Atchison et al., 1995; Buehren & Collins, 2006; Cheng et al., 2004; He et al., 2000; López-Gil et al., 2008; Ninomiya et al., 2002; Plainis et al., 2005; Radhakrishnan & Charman, 2007) as well as after in vitro ones (carried out by stretching the lens; Glasser & Campbell, 1998; Manns et al., 2007). In some cases, the authors have found a decrease in \( a_4^0 \) during accommodation, starting and finishing with positive values, but \( a_4^0 \) goes usually from a positive value for the unaccommodated eye to a negative one for accommodation states beyond 4 D. In other cases, the unaccommodated eye presents negative \( a_4^0 \), which becomes even more negative during accommodation. Thus, although the value of the \( a_4^0 \) coefficient at the starting point (relaxed eye) could be different, there is no question about the “typical decrease” (Atchison et al., 1995) of the coefficient of fourth-order SA \( (A_4 \) or \( a_4^0 \)) during accommodation in the young eye.

However, after more than one century, there is at present still no explanation of the mechanism for these changes in spherical aberrations during accommodation. The question that arises is whether or not those changes modified the accommodation response or somehow benefit human vision. Although a few examples of the potential benefit of the decrease of primary SA on the accommodation response has already been pointed out by some researchers (Buehren & Collins, 2006; Plainis et al., 2005) following experimental in vivo measurements, little has been said about how much it really benefits the accommodation response. Moreover, as far as we know, except for three studies—two in humans (Lopez-Gil, Lara, & Fernandez-Sanchez, 2006; Ninomiya et al., 2002) and another one in pigs (Roorda & Glasser, 2004)—there are no other studies that investigated the possible increase or decrease in the secondary SA, and its influence on the accommodation response has never been investigated. In the following, we will give an answer to these points by means of theoretical calculations, computer simulations, and experimental measurements.

**Methods**

**Spherical aberration change during accommodation**

Thomas Young already demonstrated several centuries ago that accommodation is achieved by means of changes in the lens (Young, 1801), in particular, a reduction of the radius of curvature of both surfaces, together with an increase in the lens thickness and a small displacement of the lens center toward the cornea. Classical and modern techniques (Glasser & Campbell, 1998; Jones et al., 2005; Koretz et al., 1987; Neider et al., 1990; Strenk et al., 1999) have shown that the largest modification corresponds to the reduction of the first radius of curvature. For instance, Le Grand’s eye model indicates that for 7 D of accommodation, the first radius of curvature goes from 10.2 to 6 mm, while the second radius only decreases from 6 to 5.5 mm and the center of the lens moves forward 0.15 mm. In vitro measurements of young subjects’ lenses also confirmed that the lens’ front surface undergoes a bigger curvature change than the posterior surface after stretching (Glasser & Campbell, 1998; Manns et al., 2007).

Since it is the physiological change of the lens that makes accommodation possible, it should also be the main factor responsible for the decrease in spherical aberration during accommodation. Then, assuming radial symmetry, we can model the lens’ front and back surfaces using a conic surface of radii \( R_{1L} \) and \( R_{2L} \) and conic constants \( k_{1L} \) and \( k_{2L} \), respectively. Appendix A shows the mathematical
calculations used to find the dependency of the eye’s wave front with \( k_{1L} \) and \( k_{2L} \) (Equation A8), as well as the change of \( A_s \) and \( B_s \) during accommodation (Equations A13 and A14).

Assuming that the pupil diameter does not change during accommodation, Equation A10 indicates that \( A_s \) should change in the negative direction (decrease) during accommodation since both surfaces of the lens contribute to it. In particular, for the front surface \( R_{1L} \) decreases (Dubbelman, van der Heijde, & Weeber, 2005), \( \beta_{1L} \) (the magnification between the plane of the lens’ front surface and the eye’s entrance pupil plane) is positive and remains nearly constant, and \( k_{1L} \) is negative (hyperbolic shape; Dubbelman et al., 2005). For the back surface, a similar situation occurs, because although the change of index of refraction is negative, the radius of curvature \( (R_{2L}) \) is also negative and so is \( k_{2L} \). (In fact, \( k_{1L} \) and \( k_{2L} \) seem to become even more negative (Dubbelman et al., 2005), while \( \beta_{1L} \) and \( \beta_{2L} \) decrease only slightly during accommodation (see Computer simulations section).) On the other hand, Equation A11 shows that \( B_s \) should change in the positive direction (increase) after accommodation, since its value depends on \( k_{1L}^2 \) and \( k_{2L}^2 \).

### Refractive state change during accommodation

Let us study the effect of the wave-front variation during accommodation upon the effective change of the refractive state defined as the inverse of the distance (vergence) of the retinal conjugate plane, that is, the target vergence required to maximize retinal image quality. Then, as hypothesis, we assume that the eye accommodates to a certain stimulus until its retinal image is maximized (López-Gil, Fernández-Sánchez, Thibos, & Montés-Micó, 2009; Tarrant, Roorda, & Wildsoet, 2010).

Conversion from wave front (Equation A8) to refractive change and accommodation can be calculated by taking into account that the refractive change produced by a defocus \( A_d \), for a pupil radius \( r_0 \) corresponds to \(-2A_dr_0^2\) (Mahajan, 1991).

Therefore, following Equation A6, the refractive change induced by the change of the defocus term generated by both surfaces of the lens is

\[
-(n_{L} - n)\left[ \frac{1}{\beta_{1L}^2 R_{1L}} - \frac{1}{\beta_{2L}^2 R_{2L}} \right].
\]  

Equation 5 corresponds to the paraxial change of accommodation generated by the front surface of the lens, that is, the change of the refractive state during accommodation caused by the front surface of the lens if high-order aberrations were not being taken into account. However, Equation A8 showed that when the eye accommodates there is not only a change in defocus but also in primary spherical aberration. It is well known that in any optical system with certain amount of primary spherical aberration, \( A_s \), the best image plane in terms of minimum variance of the wave-front aberration (RMS) will be observed in the presence of a defocus \( A_d \), such as \( A_d = -A_s \) (Mahajan, 1991). Appendix B shows an extension of this effect in the presence of primary \( (A_s) \) and secondary \( (B_s) \) spherical aberrations. In that case, the defocus necessary to minimize RMS is: \( A_d = -A_s - (9/10)B_s \). Therefore, from Equations A9 and A10, the refractive effect of primary and secondary SAs on the refractive state would correspond to

\[
\frac{(n_{L} - n)r_0^2}{4} \left[ \frac{k_{1L}}{\beta_{1L}^4 R_{1L}^3} - \frac{k_{2L}}{\beta_{2L}^4 R_{2L}^3} \right],
\]  

and

\[
\frac{9(n_{L} - n)r_0^4}{80} \left[ \frac{k_{1L}^2}{\beta_{1L}^6 R_{1L}^5} - \frac{k_{2L}^2}{\beta_{2L}^6 R_{2L}^5} \right],
\]

respectively.

The term \( W_r(r) \) in Equation A8 represents the wave-front variation induced by the cornea. Assuming that there is no any change in the cornea during accommodation, the contribution of the cornea to the change of the refractive state during accommodation would come from the change of the defocus originated by the change of cornea’s primary and secondary SAs undergone when the pupil decreases its size. Then, after Equations 5–7, and assuming that neither the indices of refraction (Hermans, Dubbelman, Van der Heijde, & Heethaar, 2008) nor \( \beta_{1L} \) and \( \beta_{2L} \) vary during accommodation (see Computer simulations section), the accommodation (refractive change from a relaxed state to a given accommodation state) can be expressed as

\[
A = -(n_{L} - n) \left[ \left( \frac{1}{\beta_{1L}^2 R_{1L}} - \frac{1}{\beta_{2L}^2 R_{2L}} \right) - \left( \frac{1}{\beta_{1L}^2 R_{1L}^2} - \frac{1}{\beta_{2L}^2 R_{2L}^2} \right) \right] + \frac{(n_{L} - n)}{4} \left[ \frac{k_{1L}^4 r_0^2}{\beta_{1L}^6 R_{1L}^5} - \frac{k_{2L}^4 r_0^2}{\beta_{2L}^6 R_{2L}^5} \right] + \frac{9(n_{L} - n)}{80} \left[ \frac{k_{1L}^4 r_0^4}{\beta_{1L}^8 R_{1L}^7} - \frac{k_{2L}^4 r_0^4}{\beta_{2L}^8 R_{2L}^7} \right],
\]  

where values without prime corresponds to the relaxed state and those with prime corresponds to the accommodated eye. The first bracket in Equation 8 is directly proportional to the curvature of the front and back surfaces of the lens \((1/R_{1L} \) and \(1/R_{2L} \); in agreement with experimental measurements; Koretz, Cook, & Kaufman, 2002) and is the main factor responsible for the refractive change of the eye during accommodation. However, since both surfaces of the lens have a negative asphericity, the
last two brackets also add some amount of accommodation (third and fourth brackets). The bracket corresponding to the effect of the primary spherical aberration (second bracket) produces a decrease in the accommodation, while the one corresponding to the secondary spherical aberration (third bracket) produces an increase in the accommodation. In total, the effect of spherical aberration results in a reduction of the accommodation since the second bracket has a larger weight than the third one.

The reduction of accommodation response produced by the decreases of the primary spherical aberration during accommodation (second bracket in Equation 8) can be understood as follows. In the relaxed eye as the myopic eye shows in Figure 1 (top), there is usually a certain positive total amount of SA4 since the cornea has a larger positive value than the one of the lens (with negative sign). That positive SA4 is compensated by some negative defocus (Figure 1, top) to minimize the RMS of the wavefront. However, in the accommodated eye (Figure 1, bottom), SA4 decreases becoming negative, and in order to attain a good retinal image, it will need an extra defocus to minimize the RMS with opposite sign to the one needed in the relaxed eye. Then, for paraxial rays, the accommodation response of the eye should be larger than it really is because the paraxial object, OP, pass from a more hypermetropic to a more myopic position with respect ORMS after accommodation (Figure 1).

To understand the effect of sixth-order spherical aberration, a similar analysis could be done, but in this case, the effect of SA6, which pass from negative to positive during accommodation (see Results section), will create a larger accommodation in the case of non-paraxial rays. However, its effect on the accommodation is much smaller than SA4 as will be seen later, and the total change of spherical aberration reduces the accommodation response of the eye.

**Computer simulations**

The exact change of spherical aberration and refraction during accommodation can be calculated by means of ray-tracing software applied to a model eye. We use Zemax-EE and a model eye with the features summarized in Table 1 to compute the change of both spherical aberrations as well as the refraction change of the eye during accommodation. The data shown in Table 1 were measured by Dubbelman, Weeber, van der Heijde, and Völker-Dieben (2002) and Dubbelman et al. (2005) and correspond to the mean values of several eyes of subjects aged around 30. They were obtained by means of in vivo measurements with a wavelength of 0.587 μm and an accommodation stimulus range between 0 and 7 D. We have assumed two types of pupil size: a 4-mm fixed pupil and a linear decrease in the pupil diameter during accommodation of 0.1 mm/D, starting from a diameter of 4.5 mm for the unaccommodated state.

\[ \beta_{1L} \text{ and } \beta_{2L}, \text{ corresponding, respectively, to the pupil magnification of the front and back surfaces of the lens, depend both on the accommodation state. However, the ray-tracing simulations have demonstrated that their value decreases very little during accommodation. For instance, when the entrance pupil was set to 4 mm (fixed value), we found: } \beta_{1L} = 1.1269 - 0.0006D (R^2 = 0.9998) \text{, and } \beta_{2L} = \]
Table 1. Parameters of the eye model used for theoretical and computer simulations. Note: $D$ indicates the stimulus accommodation in diopters. Distances are in millimeters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of the front surface of the cornea</td>
<td>7.87</td>
</tr>
<tr>
<td>Asphericity of the front surface of the cornea</td>
<td>0.85</td>
</tr>
<tr>
<td>Corneal refractive index</td>
<td>1.376</td>
</tr>
<tr>
<td>Corneal thickness</td>
<td>0.574</td>
</tr>
<tr>
<td>Radius of the back surface of the cornea</td>
<td>6.4</td>
</tr>
<tr>
<td>Asphericity of the back surface of the cornea</td>
<td>0.82</td>
</tr>
<tr>
<td>Refractive index of the aqueous humor ($n_a$)</td>
<td>1.336</td>
</tr>
<tr>
<td>Anterior chamber depth</td>
<td>2.996 - 0.036D</td>
</tr>
<tr>
<td>Entrance pupil radius</td>
<td>2 or 2.25 - 0.05 * $D$</td>
</tr>
<tr>
<td>Radius of the front surface of the lens ($R_{L1}$)</td>
<td>1 / (0.0894 + 0.0067$D$)</td>
</tr>
<tr>
<td>Asphericity of the front surface of the lens ($k_{L1}$)</td>
<td>-4.5 - 0.5$D$</td>
</tr>
<tr>
<td>Magnification between EP and L1 ($\beta_{L1}$)</td>
<td>1.1269</td>
</tr>
<tr>
<td>Refractive index of the lens ($n_L$)</td>
<td>1.4293</td>
</tr>
<tr>
<td>Lens thickness</td>
<td>3.638 + 0.043$D$</td>
</tr>
<tr>
<td>Radius of the back surface of the lens ($R_{L2}$)</td>
<td>-1 / (0.1712 + 0.0037$D$)</td>
</tr>
<tr>
<td>Asphericity of the back surface of the lens ($k_{L2}$)</td>
<td>-1.43</td>
</tr>
<tr>
<td>Magnification between EP and L2 ($\beta_{L2}$)</td>
<td>1.2945</td>
</tr>
<tr>
<td>Refractive index of the vitreous humor ($n_v$)</td>
<td>1.336</td>
</tr>
<tr>
<td>Axial length</td>
<td>24</td>
</tr>
</tbody>
</table>

1.2945 - 0.0014$D$ ($R^2 = 0.9987$), for the front and back surfaces of the lens, respectively. Thus, even when it comes to sixth-order spherical aberration, which includes a term in $\beta^{-6}$ (see Equation 8), the value of that terms does not change more than 1.6% across the abovementioned 7-D accommodation range. That is the reason why in Table 1, $\beta_{L1}$ and $\beta_{L2}$ are considered to be constants.

**In vivo ocular wave-front change during accommodation**

To measure the real change of spherical aberration during accommodation and to see its effect in the accommodation response, wave-front measurements were carried out under natural pupil and monocular conditions in 15 young subjects aged 20–38 (24 ± 6 years) having no pathologies, amblyopia, or accommodation dysfunctions. Their sphere ranged between 0.38 D and −3.06 D with a cylinder of −0.38 ± 0.25 D, which was not corrected during the experiment. The aberrometer of choice was the irx3 (Imagine Eye, France), which had a Badal system embedded inside to be able to move a target from 0.5 D beyond the subject’s far point to 9.5 D closer than that point. The stimulus varied its vergence at steps of 0.5 D, leaving 1.5 s for the subject to accommodate. The stimulus accommodation range was always larger than the subject’s accommodative amplitude. The target consisted of a polychromatic image of a balloon at the end of a road, containing a wide range of spatial frequencies in all directions, with a mean luminance of 50 cd/m².

The subject was instructed to keep the stimulus as clear as possible during the experiment. We assume that the refractive state of the subject for any position of the stimulus within the interval of vision will be such that maximizes image quality according to the minimum RMS metric.

All the subjects were informed about the study and signed an informed consent. The tenets of the Declaration of Helsinki were followed. All subjects ran at least 3 trials to get used to the methodology prior to recording the measurements.

**Results**

**Theoretical calculations and computer simulations**

Applying Equations A17 and A18 (with $[A_s]_T = 0.429 \mu m$ and $[B_s]_T = 0.081 \mu m$ for a 4.5-mm pupil) to the values shown in Table 1 permits us to obtain the theoretical change of $A_s$ and $B_s$ during accommodation. (Changes in Zernike instead of Seidel during accommodation can simply be calculated taking into account Equations 3 and 4.)

Figure 2 shows those changes for a constant pupil size of 4 mm and also when it is assumed that the pupil decreases during accommodation in the way shown in Table 1. The results yielded by the computer simulations are also presented for comparison purposes. Simulated values of $A_s$ and $B_s$ were obtained, including in Equations 3 and 4 the Zernike values calculated by the Zemax-EE software for a wavelength of 0.587 µm (Equations 3 and 4).

Assuming that in the relaxed eye accommodation response is 0 D for a certain vergence of the stimulus ($R$), accommodation response to a certain vergence of the object, $X$ (corresponding to a stimulus accommodation of $R - X$), is defined by the change of refractive state of the eye. For instance, for a myope with a refraction $R = -2 \, D$, which changes the refractive state from $-2 \, D$ (relaxed eye) to $-4 \, D$ when seeing an object placed at 20 cm ($-5 \, D$ of vergence), it will be said that for a stimulus accommodation of 3 D ($=-2 \, D - (-5 \, D)$), the eye is presenting an accommodation response of 2 D ($=-2 \, D - (-4 \, D)$).
Figure 3 shows the resulting theoretical accommodation response, obtained based on the data of Table 1 and Equation 8. The contribution of each surface of the lens is shown separately, and so are the different effects on the accommodation response of defocus and fourth- and sixth-order spherical aberrations.

**In vivo ocular wave-front changes during accommodation**

*Spherical aberration*

Figure 4 shows the values of $A_s$, $B_s$, and pupil diameter during accommodation for the 15 subjects included in the study. (Although the computer simulations and *in vivo* measurements have been obtained discretely by using a stimulus that approaches to the eye by steps of 0.5 D, a continuous line has been used to connect the results in the figures, for clarity.)

**Accommodation response**

Figure 5 shows the mean accommodation response for all subjects using Equation 8. To show the effect of both spherical aberrations, we have computed the accommodation response both including and excluding their contribution.

If we define the accommodation amplitude (AA) as the maximum range of accommodation (maximum minus minimum accommodation response), we can calculate the effect of SA4 and SA6 on the AA. Figure 6 shows the results for the 15 subjects included in the study, as well as the mean value.

**Discussion**

*Decrease in spherical aberration during accommodation*

Figures 2A and 2B show that Equations A11 and A12 correctly predict the trend suggested by the simulated ray tracing of SA4 and SA6. Nevertheless, there is no exact match, especially in the case of SA6 (Figure 2B). There are three main reasons for this small discrepancy. First, the modification undergone by the wave front after going through any of the lens’ surfaces has been approximated by the shape of the front surface multiplied by the refraction index difference before and after the surface (Equations A6 and A7). That approximation overestimates the effect of that surface on the eye’s spherical aberration, above all for high aberration values (i.e., the edges of the pupil). Second, in our theoretical calculations, we have assumed a constant value for $\beta_{1L}$ and $\beta_{2L}$, but in reality,
they are not constant (see Computer simulations section). Third, the modification of the wave front after its propagation from the second surface of the lens to the first one has been ignored, since it is expected to play a very small role.

From Equation A8, it can be seen that the main factor responsible for the decreases of SA4 and the increase in SA6 during accommodation is the front surface of the lens. In particular, for a 4-mm pupil, out of the total drop of SA4, 0.06 \( \mu m \) (86\%) is due to the lens' front surface and 0.01 \( \mu m \) (14\%) to its back surface.

From the values of Table 1 for a fixed pupil diameter of 4 mm and a stimulus accommodation of 4 D, we obtain a theoretical decrease in \( A_4 \) equal to 0.94 \( \mu m \), that is, \( \Delta A_4 = -0.99 \mu m \) (which corresponds to \( \Delta d_4^0 = -0.056 \mu m \); Figure 2A). That value is a little bit higher than the one yielded by the simulations: \( \Delta A_4 = -0.87 \mu m \) (\( \Delta d_4^0 = -0.052 \mu m \)) for the eye model based on the data of Table 1 (Figure 2A), or \( \Delta d_4^0 = -0.05 \mu m \) when using Navarro’s eye model (Navarro, Santamaría, & Bescó, 1985; López-Gil et al., 2008). Similar values have also been found experimentally by independent and previous

Figure 4. Experimental results for each individual subject regarding the change of (A) Zernike fourth-order SA, (B) Zernike sixth-order SA, and (C) pupil diameter during accommodation. Black thick line represents the mean value.

Figure 5. Calculated mean accommodative response based on the experimental data corresponding to all the subjects. Green line: only defocus \( (A_d) \) related changes are included; red line: defocus- and only fourth-order SA-related changes were included; black line: defocus-, fourth-order, and sixth-order SA-related changes were included.
studies performed by various authors: Cheng et al. (2004), $\Delta a_4^0 = -0.07 \mu m$; Plainis et al. (2005), about $\Delta a_4^0 = -0.054 \mu m$; or López-Gil et al. (2008), $\Delta a_4^0 = -0.044 \mu m$. Atchison et al. (1995) found a change of fourth-order spherical aberration in terms of defocus of $-0.34$ D, which corresponds to $\Delta a_4^0 = -0.051 \mu m$ for a 4-mm pupil. Radhakrishnan and Charman (2007) also found a value of $\Delta a_4^0 = -0.048 \mu m$, although he used a natural pupil diameter.

To our knowledge, the only published experimental data that show a significant increase in $a_6^0$ during accommodation for a fixed pupil size correspond to Ninomiya et al. (2002). Their data, although it cannot be directly compared with our calculations (Figure 2B) because the pupil diameter values are different, show a similar trend. Figure 4B clearly shows that, on average, there is an increase in $a_6^0$, which goes from a negative value to a positive one. It is interesting to point out that the effect of SA6 on the accommodation response is not negligible, as shown in Figure 6B for subjects #6, #8, and #13.

A close examination of Equations A19 and A20 reveals that when the pupil decreases ($r' < r$), we can expect a smaller change of $A_s$ and $B_s$ during accommodation, relative to the situation when the pupil size remains constant ($r' = r$). This fact is reflected in Figure 2A, where the slope of $\Delta a_4^0$ is bigger for the natural pupil condition than for the 4-mm-diameter pupil until about 5 D of stimulus accommodation, for which the natural pupil size is about 4 mm. Then, Equations A21 and A22 indicate that the change of $a_4^0$ and $a_6^0$ during accommodation will

![Figure 6](https://jov.arvojournals.org/pdfaccess.ashx?url=/data/journals/jov/933482/ on 01/11/2019)

Figure 6. (A) Amplitude of accommodation for each subject: dark blue bars show the AA without the effect of fourth- and sixth-order SAs; light blue bars show the real response including their effect. (B) Contribution of each spherical aberration to the amplitude of accommodation: fourth-order SA (light blue bars); sixth-order SA (dark blue bars). Last two bars show the mean value between subjects.

![Figure 7](https://jov.arvojournals.org/pdfaccess.ashx?url=/data/journals/jov/933482/ on 01/11/2019)

Figure 7. Experimental results for subject #4 regarding the change of pupil size (dotted line, right scale), SA4 (black line, left scale), and SA6 (gray line, right scale) during accommodation.
be bigger for larger pupils, which is in good agreement with the experimental data shown by López-Gil et al. (2008) and Radhakrishnan and Charman (2007). As an example, Figure 7 shows that when the pupil does not decrease during accommodation (stimulus accommodation between 0 and 2.5 D) or even when it increases (around 4 D of stimulus accommodation) the value of SA4 decreases rapidly.

Figure 7 also shows the typical decrease in SA4 going from positive to negative values. For this subject, beyond 8 D, SA4 reaches its lowest value and starts to increase again approaching zero. This behavior can be explained by examining Equation A9: for 8 D of stimulus accommodation, the curvature of the lens reaches a maximum, whereas the pupil diameter can still decrease. This effect could indicate that the ciliary muscle could still contract for a stimulus accommodation larger than 8 D, but that the lens cannot modify its shape any further (i.e., it cannot become more convex). A similar behavior can be seen for SA6 (Figure 4); there is an initial increase followed by an approach to zero for a stimulus accommodation above 7 D.

Although there is a large intersubject variability, the mean values shown in Figures 4A and 4B confirm the trend predicted by our calculations and previous results (Buehren & Collins, 2006; Ivanoff, 1947; Kooman et al., 1949; Plainis et al., 2005); that is, a decrease in $A_4$ and an increase in $B_4$. As indicated by other researches (Cheng et al., 2004; López-Gil et al., 2008; Plainis et al., 2005), most of the subjects have a positive $A_4$ ($a_4^0 > 0$) in the relaxed state, and a negative one beyond 3–4 D of stimulus accommodation. However, there are also some subjects for whom $A_4$ (so $a_4^0$) always remains positive (blue line in Figure 4A), and others for whom $A_4$ has a negative value across the whole accommodation range, as shown by the brown line in Figure 4A, corresponding to subject #6. In these cases where $A_4 < 0$ in the relaxed eye, the minimum SA4 was usually more negative than in the cases where the relaxed eye presented a positive SA4 value.

**Effect of spherical aberration on the accommodation response**

Figure 3 shows that, for an eye having similar features to those presented in Table 1, its accommodation response depends mainly on the defocus change produced by the front surface of the lens, which accounts for 82% of the total response. Defocus produced by the back surface of the lens accounts for 33%, while the change in SA4 produced by the front surface of the lens accounts for $-19\%$. The variation of SA6 due to the front surface of the lens accounts for 4%, while the variation of SA4 and SA6 associated to the back surface of the lens has a negligible effect on accommodation.

Figure 3 also shows that the accommodation response for 7 D of stimulus accommodation of our model eye with a 4-mm pupil is $-1.2$ D lower when the effect of SA4 is taken into account. The effect of SA6, however, increases the accommodation response but smaller (around 0.3 D). So total effect of SA in the accommodation response was 0.9 D for 7 D of stimulus accommodation, which represents a decrease of 0.13 D per diopter of stimulus accommodation.

Experimental results also confirm that tendency. Figures 5 and 6 show that for all the subjects included in the study the change of both types of spherical aberrations during accommodation results in a decrease in the accommodation response. The average decrease was 0.9 D, although decreases up to 1.3 D were observed (subject #1). On average, the effect of SA4 was about 3 times larger than that of SA6. This means that SA4 accounts for $-1.2$ D and SA6 for 0.3 D of the total AA (6.46 D). However, it is interesting to see that in some subjects (such as #1, #6, #8, and #13) the effect of SA6 could be larger than half of a diopter, resulting in increases of AA that are far from being negligible. In other subjects, such as #9 SA6 did not play any role in the accommodation response. These differences across subjects can be explained by having in mind the different geometries of the cornea and the lens and the differences in pupil size, since it decreases during accommodation. The larger the pupil, the larger the effect of SA4 and SA6 that is expected (Equations A10, A11, A19, and A20). Subject #6 is particularly interesting. He has a negative $a_4^0$ for the relaxed eye, and this value barely decreases during accommodation (see brown line at the bottom of Figure 5A). However SA4 has a clear negative impact on his accommodation, as shown in Figure 6B. However, the same subject shows a large increase in $a_6^0$.
(see brown line at the bottom of Figure 5B) resulting in a benefit on the AA of about 0.5 D (Figure 6B).

As an example of the dependency of the accommodation response on pupil size and on its variation during accommodation, we can use the same theoretical eye (Table 1) under two different pupil conditions. The first one corresponds to a 6.5-mm pupil, whose diameter decreases at a rate of 0.2 mm per diopter of stimulus accommodation (resulting in a 5.1-mm pupil diameter at the end of the 7-D range of stimulus accommodation). The second condition corresponds to a constant pupil size of 4 mm. Figure 8 shows the results.

In this case, the joint effect on refraction of SA4 and SA6 is higher for the first condition (accommodative miosis) than for the second one (fixed pupil), due to the fact that across the whole accommodation range the pupil is always larger in the first condition. In particular, in the first condition, the effects of SA4 and SA6 on refraction cause a total decrease in the accommodation response of 1.5 D at 7 D of stimulus vergence, producing a decrease in the accommodation response of 0.21 D per diopter of stimulus accommodation. However, in the second condition, SA4 and SA6 account for about 0.9 D of the accommodation response at 7 D of stimulus accommodation (0.13 D per diopter of stimulus accommodation). Furthermore, in the first condition, the joint effect of SA4 and SA6 was similar but in opposite sign to the effect of defocus produced by the second surface of the lens. The total accommodation response is far from the 1:1 response. The same tendency was found in the experimental results (Figure 5), although the effects of SA4 and SA6 are not so important as in Figure 8.

In Figures 2 and 8, we have assumed that accommodation response was zero for an object at the subject’s far point, so there is no accommodation lead. However, if we plot the accommodation response obtained using the minimum RMS using the origin of the stimulus to accommodation axis (X-axis) corresponding to the paraxial far point (maybe preferred subjectively), the accommodation response would present a lead of accommodation if the relaxed eye has a positive SA4 (as is the case in most of the subjects analyzed, see Figure 4A). Then, Figure 5 shows not only a lag but also a lead of accommodation. Figure 1 also shows schematically the presence of a lead of accommodation, which corresponds to the absolute difference in dioptries between the vergences of OP and ORM in the relaxed eye, while the lag of accommodation correspond to the same difference but in the accommodated eye (with an opposite sign than the lead). However, the presence of a lead or lag of accommodation would depend on the sign of SA4 in the relaxed eye. Thus, for SA4 = 0, no lead could be expected, while for SA4 < 0 a lag of accommodation could be expected in the whole range of accommodation, being much larger in the accommodated eye than in the relaxed eye as predicted by Equation 8.

Depending on the subject’s equivalent sphere correction (spectacles or contact lenses), accommodation response could also be modified if the refractive effect of the distance between the spectacle and the eye is not taking into account to calculate the exact value of the stimulus to accommodation.

The accommodation response caused by defocus alone is also larger in the experiment (Figure 5) than in the simulation (Figure 8). We explain this discrepancy in terms of depth of focus, different mean age between the experimental and theoretical model eyes, the approximations used in Equations A6 and A7, and probably the effect of the lens gradient index, not included in our theoretical and simulated studies (a recent investigation (Maceo et al., 2010) has pointed out the large effect on the accommodation response that the lens gradient index might have).

Contrary to what most people think, when a relatively large value of positive SA4 is added to the eye, the eye becomes more hyperopic, while adding a negative SA4 to the eye, its refraction will be shifted in the myopic direction. This effect has been observed by many researchers after adding some amount of Zernike SA4 by means of contact lenses or deformable mirrors (Applegate, Marsack, Ramos, & Sarver, 2003; Benard, López-Gil, & Legras, submitted for publication; Rocha, Vabre, Château, & Krueger, 2009). This is because the refraction change induced by SA4 is mainly due to the amount of defocus (the term in \( r^4 \)) that the polynomial includes to balance the term in \( r^4 \) (Equation 2), thus minimizing the RMS. However, that only happens for relatively large amounts of induced \( a_4^0 \). For small values of \( a_4^0 \) (below approximately 0.1 \( \mu m \)), the visual system does not have its refraction modified, as has been pointed out recently by Cheng, Bradley, Ravikumar, and Thibos (2010). When Seidel’s primary spherical aberration is added (Seidel SA, which is similar to an “unbalanced Zernike spherical aberration”), the opposite effect is expected. That is, low values of Seidel SA will modify the refraction, while large values will hardly modify it. In particular, positive small amounts of Seidel SA induce a negative defocus (hyperopic eye) while a negative small amount of Seidel SA induces a positive defocus (myopic eye).

Our theoretical results (Equation A8) suggest that there is a relative small decrease in Seidel SA during accommodation. The average change of SA4 was 0.32 \( \mu m \) (0.06 \( \mu m \) for SA6), which is larger than 0.1 \( \mu m \), indicating that probably the refraction change produced by the effect of SA4 (Figures 3 and 8) could be overestimated. That is probably the main discrepancy between the experimental accommodation response (Figure 5) and the calculated one (Figure 8). However, even in the case that only half of that SA4 was really used to change the refraction state of the eye, it also causes the decrease in AA by about 0.7 D (for a mean pupil size of 5 mm), which is a large value (10% of the 1:1 response). Moreover, we should keep in
mind that the change of SA4 during accommodation is continuous, so its effect could be different from the case where SA4 is added instantaneously, as in the studies previously (Benard et al., submitted for publication; Rocha et al., 2009) undertaken.

Our main hypothesis maintains that the eye is accommodating to maximize the image quality. Assuming that image quality is maximized by minimizing the RMS of the wave front, Equation 8 predicts a decrease in the accommodation response with respect the case where the eye would use only the paraxial rays (insensitive to SA effects). This explains the presence of “errors” in the accommodation response as shown in Figures 3 and 8. However, we do not really know if the visual system minimizes the RMS during accommodation when the stimulus light is traveling from the stimulus toward the retina (first pass). Nevertheless, the light traveling from the retina to the apparatus of measurement (second pass) is also affected by the change of the spherical aberration during accommodation. Thus, depending on the methodology used to measure the refractive state of the eye by the apparatus, accommodation response could also show a lead and/or lag of accommodation due to the second pass. The measured amount of accommodation lead or lag will then depend on the depth of field, the amount of the spherical aberration presented in the relaxed eye, its changes during accommodation, and the image quality metric used by the visual system and the apparatus used in the measurements.

Conclusions

A decrease in fourth-order and an increase in sixth-order spherical aberration during accommodation are due to the hyperbolic shape of the surfaces of the human lens. Depending on the objective method used to measure the accommodation response, this variation of spherical aberration—and above all the one undergone by the fourth-order spherical aberration at the front surface of the lens—has an important effect on the retinal image quality, modifying the position of the image quality plane. In case that minimum RMS is used as the image quality criteria, the changes of fourth-order spherical aberration produce a decrease while the changes of sixth-order spherical aberration increase the accommodation response. Total contribution of fourth- and sixth-order spherical aberrations reduces the accommodation response about 1/7 of diopter per diopter of proximity of the stimulus.

Thus, excluding the small depth of field of the eye, the apparent accommodation error in the accommodation response of the object in the interval of vision where the eye is getting a good (eventually the best) image is not really an “error” but a consequence of the strategy used by the visual system and the apparatus of measurement to select the best image plane that can be affected by the presence of spherical aberration in the relaxed eye and its changes during accommodation.

Appendix A

The front surface of the lens can be described using a conic surface that, in Cartesian coordinates, can be expressed as

\[(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = R_{1L}^2 - (k_{1L} - 1)z^2, \tag{A1}\]

where \(R_{1L}\) and \(k_{1L}\) are the radius of curvature and the conic constant of the surface, respectively (Figure A1).

If the center of the conic surface has coordinates \(x_0 = y_0\) and \(z_0 = R_{1L}\), we can rewrite Equation A1 in polar coordinates as

\[r_{1i}^2 + k_{1L}z^2 - 2R_{1L}z = 0, \tag{A2}\]

where \(r_{1i}\) represents the radial coordinate at the iris plane.

Figure A1. Front surface of the lens represented as a conic curve with its apex at the origin of coordinates.
Solving Equation A2 for \( z \), we have

\[
R_{1L} \left[ 1 - \sqrt{1 - k_{1L} \left( \frac{r_{1i}}{R_{1L}} \right)^2} \right]
\]

\[ z = \frac{R_{1L} \left( 1 - \sqrt{1 - k_{1L} \left( \frac{r_{1i}}{R_{1L}} \right)^2} \right)}{k_{1L}}, \tag{A3} \]

where only the negative root has been taken into account in order to have the vertex of the surface at the origin of coordinates (see Figure 1). Now we can rewrite Equation A3 as a Taylor-series expansion around the origin, taking into account that \( \sqrt{1 + x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^2}{16} - \ldots \). \tag{A4}

As a result, following an expansion up to the sixth order in \( r_{1i} \), Equation A3 can then be expressed as (see Roorda & Glasser, 2004 for a similar result for a calculation up to fourth order)

\[
z(r_{1i}) = \frac{r_{1i}^2}{2 \cdot R_{1L}} + \frac{k_{1L} r_{1i}^4}{8 R_{1L}^3} + \frac{k_{1L}^2 r_{1i}^6}{16 R_{1L}^5}.	ag{A5}\]

We can now think of a point light source located on the fovea that produces light. To exit the eye, this light will have to go through the eye, as many of the aberrometers work today. When the light is passing through the front surface of the lens, it will modify the wave front by adding

\[
W_{1L}(r_{1i}) = (n_L - n_a) z(r_{1i}), \tag{A6}\]

where \( n_L \) and \( n_a \) represent the index of refraction of the aqueous humor and the lens, respectively, already taking into account the sign of the wave front when it is getting out of the eye (Rocha et al., 2009).

The wave front generated by the back surface of the lens can also be analyzed using a similar theoretical calculation. In that case, we will obtain

\[
W_{2L}(r_{2i}) = (n_v - n_L) z(r_{2i}), \tag{A7}\]

where \( n_v \) represent the index of refraction of the vitreous.

The front surface of the lens is located at the stop plane of the eye. The back surface is not located exactly at the stop plane of the eye, but the change in the wave front after propagation from the second surface to the front surface should be small and may be neglected. Then, taking into account that there is certain magnification between both surfaces of the lens and the entrance pupil plane (where aberrations are measured), which correspond to \( \beta_{1L} = r_{1i}/r_{1L} \) and \( \beta_{2L} = r_{2i}/r_{2L} \) for the front and back surfaces, respectively, the total radial dependence of the wave front at the eye’s entrance pupil would be, after Equation A5:

\[
W(r) = W_C(r) + W_{1L}(r) + W_{2L}(r)
\]

\[
= W_C(r) + \frac{(n_L - n_a)^2}{2} \left[ \frac{1}{\beta_{1L}^2 R_{1L}} - \frac{1}{\beta_{2L}^2 R_{2L}} \right]
\]

\[
+ \frac{(n_L - n_a)^4}{8} \left[ \frac{k_{1L}}{\beta_{1L}^4 R_{1L}^3} - \frac{k_{2L}}{\beta_{2L}^4 R_{2L}^3} \right]
\]

\[
+ \frac{(n_L - n_a)^6}{16} \left[ \frac{k_{1L}^2}{\beta_{1L}^6 R_{1L}^5} - \frac{k_{2L}^2}{\beta_{2L}^6 R_{2L}^5} \right], \tag{A8}\]

where \( W_C(r) \) represents the radial contribution of the cornea, up to the sixth order \( (n = 2, 4, \) or 6), and where we have assumed that \( n_a = n_v = n \). The term related to the first bracket corresponds to defocus:

\[
[A_d]_L = \frac{(n_L - n_a)^2}{2} \left[ \frac{1}{\beta_{1L}^2 R_{1L}} - \frac{1}{\beta_{2L}^2 R_{2L}} \right]. \tag{A9}\]

The terms associated with the second and third brackets in Equation A8 are the primary \( (A_s) \) and secondary \( (B_s) \) spherical aberrations generated by the lens, respectively:

\[
[A_s]_L = \frac{(n_L - n_a)^4}{8} \left[ \frac{k_{1L}}{\beta_{1L}^4 R_{1L}^3} - \frac{k_{2L}}{\beta_{2L}^4 R_{2L}^3} \right], \tag{A10}\]

\[
[B_s]_L = \frac{(n_L - n_a)^6}{16} \left[ \frac{k_{1L}^2}{\beta_{1L}^6 R_{1L}^5} - \frac{k_{2L}^2}{\beta_{2L}^6 R_{2L}^5} \right]. \tag{A11}\]

In Zernike expansion, aberration coefficients depends on the pupil radius (see Equation 2), thus the variation of \( a_d^0 \) would also be affected by the term \( W_C \) (see Equation A8). In that case,

\[
[A_s]_T = [A_s]_C + [A_s]_L = [A_s]_C \left( \frac{r_a^4}{r^4} \right) + [A_s]_L^C
\]

\[
= ([A_s]_T - [A_s]_L) \left( \frac{r_a^4}{r^4} \right) + [A_s]_L^C, \tag{A12}\]

where the prime indicates the values after accommodation, and \( [A_s]_T \) and \( [A_s]_C \) are the values of the fourth-order spherical aberration of the whole eye and that
corresponding to the cornea, respectively. Thus, the change of the total fourth-order spherical aberration during accommodation is given by
\[
\Delta[A_s]_T = [A_s]'_T - [A_s]_T = [A_s]_T \left( \frac{r^4}{r^4} - 1 \right) + [A_s]'_L - [A_s]_L \left( \frac{r^4}{r^4} - 1 \right),
\]
(A13)

Similarly, the variation in the eye’s sixth-order spherical aberration, \([B_s]_T\), would be
\[
\Delta[B_s]_T = [B_s]'_T - [B_s]_T
\]
\[
= [B_s]_T \left( \frac{r^6}{r^6} - 1 \right) + [B_s]'_L - [B_s]_L \left( \frac{r^6}{r^6} - 1 \right).
\]
(A14)

If we do not take into account pupil miosis during accommodation \((r' = r)\); for instance, by using an artificial pupil in front of the eye), the last equations can be rewritten as
\[
\Delta[A_s]_T^\varnothing = [A_s]'_L^\varnothing - [A_s]_L^\varnothing,
\]
(A15)
\[
\Delta[B_s]_T^\varnothing = [B_s]'_L^\varnothing - [B_s]_L^\varnothing,
\]
(A16)

where the symbol “\(\varnothing\)” indicates that the values correspond to a constant pupil size.

However, when pupil miosis (natural pupil) is taken into account, Equations A13 and A14 can be rewritten as a function of the change undergone by the spherical aberration for a constant pupil size (Equations A15 and A16), as follows:
\[
\Delta[A_s]_T = \Delta[A_s]_T' \left( \frac{r'}{r} \right)^4 + [A_s]_T \left( \frac{r^4}{r^4} - 1 \right),
\]
(A17)
\[
\Delta[B_s]_T = \Delta[B_s]_T' \left( \frac{r'}{r} \right)^6 + [B_s]_T \left( \frac{r^6}{r^6} - 1 \right).
\]
(A18)

Assuming that \(\beta_{1L}\) and \(\beta_{2L}\) do not change during accommodation, this leads to
\[
[A_s]_L^\varnothing = [A_s]'_L \left( \frac{r}{r} \right)^4,
\]
(A19)
\[
[B_s]_L^\varnothing = [B_s]'_L \left( \frac{r}{r} \right)^6.
\]
(A20)

In terms of a Zernike polynomial expansion, Equations A10 and A11 would be (Mahajan, 1991)
\[
[a^0_4]_L = \frac{(m_L - n)r_0^4}{48\sqrt{5}} \left[ \frac{k_{1L}}{\beta_{1L}^4 R_{1L}^4} - \frac{k_{2L}}{\beta_{2L}^4 R_{2L}^4} \right],
\]
(A21)
\[
[a^0_6]_L = \frac{(m_L - n)r_0^4}{320\sqrt{7}} \left[ \frac{k_{2L}}{\beta_{1L}^6 R_{1L}^6} - \frac{k_{2L}}{\beta_{2L}^6 R_{2L}^6} \right],
\]
(A22)

and Equations A13–A20 remain the same except changing \(A_s\) by \(a^0_4\) and \(B_s\) by \(a^0_6\).

**Appendix B**

The defocus \(A_d\) necessary to balance the effect of the presence of primary and secondary spherical aberrations, in terms of minimum RMS, will be obtained for an \(A_d\) value that minimizes the function \(\langle W(\rho)^2 \rangle - \langle W(\rho) \rangle^2\) with
\[
\langle W(r)^n \rangle = \frac{1}{\pi} \int_0^1 \int_0^{2\pi} [A_d \rho^2 + A_s \rho^4 + B_s \rho^6] \rho d\rho d\vartheta.
\]
(B1)

Thus,
\[
\langle W(\rho)^2 \rangle - \langle W(\rho) \rangle^2
\]
\[
= \frac{1}{\pi} \int_0^1 \int_0^{2\pi} \left[ A_d \rho^2 + A_s \rho^4 + B_s \rho^6 \right] \rho d\rho d\vartheta
\]
\[
- \left[ \frac{1}{\pi} \int_0^1 \int_0^{2\pi} [A_d \rho^2 + A_s \rho^4 + B_s \rho^6] \rho d\rho d\vartheta \right]^2
\]
\[
= \left( \frac{9}{112} \right) B_s^2 + \left( \frac{4}{45} \right) A_s^2 + \left( \frac{1}{12} \right) A_d^2 + \left( \frac{1}{6} \right) B_s A_s + \left( \frac{3}{20} \right) B_s A_d + \left( \frac{1}{6} \right) A_s A_d.
\]
(B2)

The value \(A_d\) that minimizes Equation B2 is obtained using
\[
\frac{\partial (\langle W(\rho)^2 \rangle - \langle W(\rho) \rangle^2)}{\partial A_d} = \left( \frac{1}{6} \right) A_d + \left( \frac{3}{20} \right) B_s + \left( \frac{1}{6} \right) A_s = 0.
\]
(B3)
So the defocus necessary to balance the primary and secondary spherical aberrations generated by the eye during accommodation will be

$$A_d = -\left(\frac{9}{10}\right)B_s - A_s. \quad (B4)$$

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