Supplementary Material: Neuronal competition models

1. Double-well potential model: We follow a model similar to that developed in (Moreno-Bote et al., 2007). The dynamics of the variable $r$ in the double-well potential is described by eq. (1) in the main text

$$\tau \frac{d}{dt} r = -4r(r^2 - 1) + I_A - I_B + n(t) ,$$  \hspace{1cm} (1)

where $\tau=10\text{ms}$ is the timescale of the dynamics, the currents $I_A$ and $I_B$ measure the stimulus strength in favor of percept A or B, respectively, and $n(t)$ is a noise term that obeys

$$\frac{d}{dt} n = -\frac{n}{\tau_s} + \sigma \frac{2\xi}{\sqrt{\tau_s}}(t) ,$$  \hspace{1cm} (S1)

where $\tau_s=100\text{ms}$, $\xi(t)$ is a white noise process with zero mean and unit variance $\langle \xi(t)\xi(t') \rangle = \delta(t-t')$, where $\delta(t-t')$ is the delta function, and $\sigma=0.9$ is the amplitude of the noise. With this definition, the noise has zero mean and variance $\sigma^2$. This noisy input models the near random input spike trains typically found in cortex (see e.g. (Moreno-Bote and Parga, 2004)).

2. Rate-based models: Here we use two different rate-based models. They differ substantially in their architecture.

2.1. Competition neuronal model with direct cross-inhibition: The evolution of the populations firing rates $r_A$ and $r_B$ of populations A and B is determined by the following coupled differential equations (Shpiro et al., 2007)

$$\tau \frac{d}{dt} r_A = -r_A + f(\alpha r_A - \beta r_B + I_A - a_A + n_A)$$

$$\tau \frac{d}{dt} r_B = -r_B + f(\alpha r_B - \beta r_A + I_B - a_B + n_B).$$  \hspace{1cm} (S2)
Here $\tau = 10\, ms$ is the time constant in which the firing rate relaxes to its steady state. The function $f(x)$ is the input-output function and determines the steady state firing rate given the inputs received by the population, and it is taken to be a sigmoid function

$$f(x) = \frac{1}{1 + e^{-(x-\theta)/k}} ,$$  

(S3)

with threshold $\theta = 0$ and $k = 0.1$. Recurrent excitatory connections in each population with strength $\alpha = 0.2$ provide positive inputs. Cross-inhibition between the populations has strength $\beta = 0.8$. External inputs (such as contrast of one image) are modeled as linear currents into the neurons in the populations A and B, with values $I_A$ and $I_B$ respectively. The intensity of the external inputs correlates with the external information in the stimulus that supports one or the other interpretation. Note that with the value of the threshold given above ($\theta = 0$), positive values of the external inputs correspond to suprathreshold stimulation, i.e., input currents whose values are above threshold, while negative values of the external inputs, which are also allowed, correspond to subthreshold stimulation, i.e., input currents whose values are below threshold. Firing rate adaptation is modeled as an hyperpolarizing current in each population, $\alpha_i$ ($i = A,B$), which obeys the equation

$$\tau_a \frac{da_i}{dt} = -a_i + \gamma r_i ,$$  

(S4)

where $\tau_a = 2\, s$ is the timescale of adaptation and $\gamma = 0.4$ is its maximum amplitude. We introduce noisy terms in the inputs of each population, $n_i$ ($i = A,B$), which follow an Ornstein-Uhlenbeck process (Risken, 1989) as in eq. (S1), with zero mean and deviation $\sigma = 0.06$.

### 2.2. Attractor model with indirect cross-inhibition

Here we follow the equations of an attractor model for perceptual bistability presented in ((Moreno-Bote et al., 2007); see their Fig. 3B and the dynamics equations in their Appendix B). The activity of the excitatory population $i = A,B$, $r_i$, is described by the equation

$$\tau \frac{d r_i}{dt} = - r_i + f(\alpha r_i - \beta r_{i,inh} + I_i - a_i + n_i) ,$$  

(S5)
with $\alpha=0.75$, $\beta=0.5$. Here $f$ is the input-output curve defined as in eq. (S3) with threshold $\theta=0.1$ and $k=0.05$. The noise terms and adaptation currents are described by eqs. (S1,S4), with $\sigma=0.03$, $\tau_a=2s$ and $\gamma=0.1$ (producing weak adaptation). Inhibition is generated by a local inhibitory population with fast response to its inputs and modeled as $r_{i,inh} = (r_{pool} + \eta r_i)^2$, where $\eta=0.5$. Inhibition receives inputs from the local excitation, and also from a global excitatory pool with rate $r_{pool}$ governed by the equation $r_{pool} = \left[ \phi (r_A + r_B) + I_A + I_B \right]^+$, where $[.]^+$ denotes linear thresholding, and $\phi=0.5$.

2.3. Input gain normalization: The models presented above can be modified by adding a preprocessing stage in their external inputs $I_A$ and $I_B$ of the two populations. This is done by replacing the input $I_i$ ($i=A,B$) to all populations in the network by $I'_i$, defined in eq. 2 (Methods). Since the currents $I'_i$s into the populations are normalized versions of the $I_i$s, we refer to the former as gain normalized currents. We used $s=1.55$ in the competition neuronal model with direct inhibition (section 2.1) and $s=0.182$ in the attractor model with indirect inhibition (section 2.2). The scaling coefficient was introduced in order to produce similar alternation rates for the models with and without input gain normalization. The qualitative results do not depend on the values of these parameters.

3. Numerical procedures: The dynamical equations for the energy and rate-based models were integrated using Euler’s method with time step $\delta t=0.1$ ms. The dominance durations for each percept in the energy model are defined by the amount of time in which the variable $r$ is below (or above) $r=0$. For the rate-based model, a transition occurs when the firing rate becomes larger (or smaller) than the firing rate of the other population. Energy and rate-based models typically run for $10^6$ s (model time), generating around $10^5$ durations for each percept. Means in all the plots are computed from the time series generated with these long simulations and error bars correspond to standard errors of the means. We used C custom code scientific library for random number generator to simulate the models and Matlab to analyze and plot the data.
Fig SM1. The symmetry of the mean dominance duration vs. fractions of dominance holds for individual subjects.

From left to right, fraction of dominance vs. contrast of one image B, mean dominance durations vs. contrast, and mean dominance durations vs. fraction of dominance of image B for two representative subjects.
Fig SM2. Symmetry of the mean dominance duration vs. fractions of dominance for each eye.

From left to right, fraction of dominance vs. contrast of the image presented to one eye, mean dominance durations vs. contrast, and mean dominance durations vs. fraction of dominance of one eye when the contrast is changed in the right (top row) and in the left eye (bottom row). Data is averaged across subjects.
**Fig SM3.** The symmetry of the mean dominance duration vs. fractions of dominance does not depend on the cutoff durations.

From left to right, fraction of dominance vs. contrast of image B, mean dominance durations vs. contrast, and mean dominance durations vs. fraction of dominance of image B for two different cutoff durations.

**Bibliography**


