Human observers are able to localize the relative position of objects defined by Gaussian variations in either luminance (1st order) or contrast (2nd order). However, positional sensitivity is significantly poorer for 2nd- than for 1st-order stimuli. These judgments require the visual system to construct a representation of pertinent variation from a number of individual retinal samples—a process known as interpolation. We compared 1st- and 2nd-order interpolation mechanisms to examine whether differences in this process underlie differences in positional sensitivity. Observers were required to judge the relative position of two vertically separated 1st- or 2nd-order Gaussian distributions. The distributions were discretely sampled, and both sample number and separation were systematically varied. Results showed that with fixed sample separation (1.9 or 7.7 arcmin), optimum localization was obtained with a minimum of 4–6 samples, for both 1st- and 2nd-order Gaussian distributions. The distributions were discretely sampled, and both sample number and separation were systematically varied. Results showed that with fixed sample separation (1.9 or 7.7 arcmin), optimum localization was obtained with a minimum of 4–6 samples, for both 1st- and 2nd-order Gaussian distributions. When sample number is maintained above this critical value, marked changes in sample separation (0 to 9 arcmin) had relatively little impact on thresholds for both 1st-order and 2nd-order stimuli. These results suggest that both 1st-order and 2nd-order interpolation mechanisms are limited by sample number rather than separation, and require a similar number of samples to mediate positional judgments.

Keywords: spatial localization, interpolation, 1st-order, 2nd-order

The fact that ‘sub-receptor’ positional thresholds are demonstrable indicates that the pooling of information across receptor space does not result in a dramatic loss of relevant positional information. Indeed, the visual system seems capable of constructing a surprisingly accurate representation of the stimulus from its individual samples, and then extracting some general feature of this distribution in order to make positional judgements. There are a number of candidate features that could be exploited, but the general consensus is that the visual system extracts the centroid of the neural representation of the stimulus distribution (Badcock & Westheimer, 1985; Levi & Westheimer, 1987; Morgan & Aiba, 1985; Whitaker & McGraw, 1998; Whitaker, McGraw, Pacey, & Barrett, 1996). This raises the intriguing proposition that information could potentially be removed from the image without any detriment to visual performance.

The ability of the human visual system to localize sparse representations of extended distributions was investigated in an elegant series of experiments by Morgan and Watt (1982). They presented a periodic visual pattern, which was discretely windowed by a number of samples and manipulated the spatial separation between samples. The visual stimulus to be localized then takes on the appearance of a series of vertical bars, in which the exact luminance value of the underlying profile is represented for that particular horizontal position of the pattern. The subject’s task was then to localize the underlying distribution, rather than the samples themselves, relative to an identical positional reference. Morgan and Watt found that positional thresholds were surprisingly immune to increases in sample separation that were up to an order of magnitude greater than the localization threshold itself. However, beyond a critical sample separation (~3 arc minutes), positional sensitivity rapidly declined, suggesting that the interpolation mechanism responsible for reconstituting the underlying distribution operates over a fixed spatial extent. However, in this particular experiment, as the separation between samples was increased, the effective number of samples available to any interpolation mechanism was reduced as samples dropped off the edge of the underlying profile. It therefore becomes difficult to ascertain whether sample separation, or the number of available samples, cause the dramatic rise in positional thresholds. This issue was revisited by Kontsevich and Tyler (1998), who used a similar localization task and showed that provided at least 3 to 4 samples are present, positional accuracy for luminance-defined objects was uncompromised.

The visual system is adept at not only localizing objects defined by luminance variations (1st-order), but also those defined by variations in contrast, motion or texture—often referred to as 2nd-order attributes (Badcock & Derrington, 1985; Cavanagh & Mather, 1989). Luminance variations are analyzed by linearly summatung neurons in the primary visual cortex that signal differences in average luminance between the excitatory and inhibitory regions of their receptive field (Movshon, Thompson, & Tolhurst, 1978; Shapley & Lennie, 1985). However, objects defined by variations in contrast, for example, are invisible to linear neurons since they produce no net difference in signal between the excitatory and inhibitory sub-regions of the receptive field. Instead, the local contrast variations are first analyzed by linear neurons tuned to relatively high spatial frequencies, whose output is then subjected to a non-linear step such as rectification. The rectified output is then conveyed to a second-stage linear filter tuned to a much coarser spatial scale. Due to the lack of overlap in filter size before and after rectification, 1st-order information cannot pass through this cascade. This separate 2nd-order processing stream of filter-rectify-filter has been identified in the striate cortex of cats (Zhou & Baker, 1993) and is also supported by psychophysical (Badcock, Clifford, & Khun, 2005; Badcock & Khun, 2001; Chubb & Sperling, 1988; Edwards & Badcock, 1995; Lin & Wilson, 1996; McGraw, Levi, & Whitaker, 1999; Morgan & Baldassi, 1997; Morgan, Mason, & Baldassi, 2000; Whitaker, McGraw, & Levi, 1997) and brain imaging studies in humans (Dumoulin, Baker, Hess, & Evans, 2003; Larsson, Landy, & Heeger, 2006). The ability of the visual system to relatively localize 2nd-order stimuli is considerably poorer than that for 1st-order stimuli (Volz & Zanker, 1996; Whitaker & McGraw, 1998). One possibility is that this performance gap may reflect critical differences in the underlying interpolation mechanisms that reconstruct the neural representation of each intensity distribution, prior to feature extraction and subsequent localization. Indeed, Sukumar and Waugh (2007) suggest the size of the interpolation zone may be larger for second-order stimuli.

In the present study we examine whether the localization of sampled distributions is limited by sample separation, as suggested by Morgan and Watt (1982), or by sample number, as proposed by Kontsevich and Tyler (1998). More importantly, we then go on to compare the characteristics of spatial interpolation mechanisms for both 1st-order and 2nd-order visual processing.

**Methods**

**Stimuli**

Stimuli were generated and controlled using MATLAB 6.0, and presented on a 17” Sony Trinitron 17SeII color monitor. The non-linear luminance response of the display was linearized using the inverse function of the luminance response as measured with a Cambridge Research Systems (CRS) Optical. The mean luminance of the display was 54.2 cd.m⁻², with a frame rate of 120 Hz and CIE 1932 xy chromaticity coordinates of x = 0.273 and y = 0.283. The host computer was an Edsys (Pentium II) PC
housing a CRS VSG2/4 graphics card. A chin and forehead rest was used to maintain the viewing distance at 2 meters.

### Continuous distributions

The stimuli were presented near the center of a uniform gray screen (54.2 cd.m\(^{-2}\)) within a rectangular vertical aperture (2.74° × 2.74° with upper and lower segments separated by 6.42 arcmin), and were windowed by a Gaussian function in the horizontal direction (see Figures 1a–1d). The 1st-order (luminance-defined) stimuli were created by adding random noise (N(x)) to the Gaussian luminance profile (G(x)). The mathematical description of these stimuli in the horizontal direction is given by:

\[
L_m(1 + G(x) + bG(x)N(x) + N_1(x,y)) + mbN_2(x,y),
\]

where \(G(x)\) and \(N(x)\) are as defined below. This then clearly shows the components.

\[L_m = \text{mean luminance}\]

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\[
L_m = \text{mean luminance}
\]

\[
G(x) = \text{Gaussian luminance profile}
\]

\[
N(x, y) \text{ is binary noise (±1) scaled by } b = .2 \text{ so that the background noise level is 0.2}
\]

\[
bG(x)N(x) \text{ is a noise scaler to null the change in Michelson contrast that occurs when the local mean luminance level changes.}
\]

\[
mbN_2(x, y), \text{ which is a random noise mask (m = .2) that is laid down to cover any residual systematic changes in local mean luminance, i.e. to remove any local second order signals.}
\]

The 2nd-order (contrast-defined) stimuli were created by multiplying the random noise field with a Gaussian envelope. The mathematical description of these stimuli in the horizontal direction is given by:

\[
L_m(1 + (G(x) + b)N_1(x,y) + mbN_2(x,y)),
\]

where \(G(x)\), \(N(x)\), m and b are as defined above and b = 0.2 is added to \(G(x)\) so that the contrast of the surrounding background texture maintains an average contrast of b when the Gaussian declines to zero.

Where \(G(x) = 0.5\exp(-x^2/2\sigma^2); N(x) = 2(\text{round}(\text{rand}(x)) \pm 1. \text{ x denotes the horizontal pixel location, } \sigma \text{ is the standard deviation of the Gaussian envelope, round transforms a rational number to an integer, } \text{rand generates a random number between 0 and 1, and } \text{b and m are constants assigned the same values as in Equation 1. The contrast of the Gaussian was 0.5 in both Equations 1 and 2 and this value controls modulation depth in both cases.}

The array of noise elements, which consisted of 1-by-1 pixel squares of diameter 0.011 deg. were randomly assigned a value of either ±1. A number of these images were independently created and displayed as a dynamic sequence at a temporal frequency of 12 Hz. A new dynamic sequence was generated on each trial. The screen resolution was set well below maximum (800 × 600 pixels) to minimize adjacent pixel non-linearities (Klein, Hu, & Carney, 1996) The multiplicative noise term in the 1st-order stimulus was chosen such that the local Michelson contrast remained approximately constant across the stimulus profile, and then the random noise field mask was added ensuring no systematic contrast-defined cues remained. Similarly, the additive noise in the 2nd-order stimulus was chosen such that the local Michelson contrast varied in an equivalent way in this stimulus type.

### Sampled distributions

These were constructed by a discrete sampling of the continuous Gaussian distribution. Each rectangular sampling window was a single pixel wide (0.64 arcmin), and extended to the full vertical length of the window.
(see Figures 1c and 1d). Both the sample number and the spacing between samples were systematically manipulated.

**Procedures**

Observers were asked to perform a single interval, forced choice Vernier alignment task, in which the horizontal position of the upper distribution had to be judged with reference to the lower distribution (see Figures 1a and 1b). The vertical separation between each of the distributions was 0.107 deg. The upper distribution could be presented at any one of 7 offsets, equally spaced around an alignment position determined by an initial method of adjustment. A step size of 0.001 deg. between each of the 7 offsets typically produced an appropriate range of responses varying from approximately 100% rightwards to 100% leftwards. Stimuli were presented within a rectangular temporal window of 500 ms duration. The results of the first 20 trials were discarded to allow subjects to familiarize themselves with the task. Following these, 40 trials were presented at each of the 7 offsets and the proportion of “rightward” responses was calculated for each offset. The resulting data were fitted with a logistic function of the form

$$y = \frac{100}{1 + e^{-\frac{(x - \mu)}{\theta}}}$$

where $\mu$ is the offset corresponding to the 50% level on the psychometric function (offset corresponding to perceived alignment) and $\theta$ provides an estimate of alignment threshold (half the offset between the 27% and 73% levels on the psychometric function approximately). The window containing both distributions was randomly jittered from trial-to-trial (0–0.14 deg.) to prevent the subjects using the edges of the screen as an alternative positional reference. Similarly, for the sampled distributions, the location of the samples relative to the underlying distribution was randomly jittered from trial-to-trial over a range equal to ±1 sample separation. This avoids a constant cue, such as the peak of the distribution, being used to make the localization judgement.

In order to avoid the potential confounds of previous studies, we wanted to keep the sample separation fixed, but at the same time systematically vary the number of sample windows. Therefore, to obtain a fixed sample separation while increasing the number of samples present, we increased the width of the underlying Gaussian distribution in proportion to sample number. Prior to this, we measured the effects of increasing the width, or standard deviation, of the Gaussian envelope on thresholds. By quantifying this relationship it is possible to re-scale the individual data to account for the effect of changing distribution width (see Appendix A). This procedure allows us to examine the influence of sample number, at a fixed sample separation, in isolation.

**Subjects**

Four subjects participated in the experiments. All were experienced psychophysical observers and undertook several practice sessions prior to data collection. Subject SK was naive as to the purposes of the experiment. All subjects had normal or corrected to normal visual acuity and wore their appropriate refractive correction during data collection. Data were collected under conditions of dim room illumination.

**Results**

In Figures 2a and 2b alignment thresholds are plotted for four subjects as a function of the standard deviation of the Gaussian envelope for both 1st-order and 2nd-order stimuli. Error bars represent ±1 standard deviation of the parameter value.
stimuli. These data show that thresholds, for both types of visual structure, increase linearly with increasing Gaussian width, a result that is in keeping with previous reports (Hess & Hayes, 1994; Toet & Koenderink, 1988; Whitaker, Bradley, Barrett, & McGraw, 2002). For 1st-order stimuli, a doubling of the standard deviation results in an approximately two-fold increase in threshold. However, 2nd-order stimuli show increased levels of threshold elevation with increasing Gaussian width, as evidenced by the slightly steeper slopes in Figure 2b ($t(3) = 3.5, p = .04$, Cohen’s $d = 1.05$). This increased rate of threshold elevation may be indicative of additional noise sensitivity either at the initial stage, which is tuned to higher spatial frequencies, or at the rectification step in the 2nd-order processing cascade. The intercepts of the fitted lines are also higher for 2nd-order stimuli ($t(3) = 5.86, p = .01$, Cohen’s $d = 3.15$), the combination producing alignment thresholds for 2nd-order stimuli that were significantly poorer than their 1st-order counterparts across the range of Gaussian widths tested (Volz & Zanker, 1996).

Vernier alignment thresholds were then measured for sampled Gaussian distributions, where the sample windows were separated by either 1.9 arcmin or 7.7 arcmin. These two particular separations were chosen because they fall in the different performance regions originally described by Morgan and Watt (1982). At a sample separation of 1.9 arcmin, their positional thresholds were similar to that of a continuous distribution, while at 7.7 arcmin separation, elevated thresholds were reported. The results for the smaller sample separation of 1.9 arcmin are shown in Figures 3a and 3c and thresholds for the larger sample separation are presented in Figures 3b and 3d. In both cases a u-shaped function is obtained. For the smaller sample separation (1.9 arcmin) optimum alignment thresholds, for both 1st- and 2nd-order stimuli, are obtained with a sample number of around six. Sample numbers larger or smaller than this value result in reduced positional accuracy. Similarly, at the larger sample separation (7.7 arcmin) thresholds for both types of stimuli are lowest with between six and eight sample windows, again deteriorating either side of this range.

The re-scaling process described earlier to account for the effect of changing the width of the underlying distribution was carried out for each observer, at both sample separations and the results are presented in Figures 4a–4d. The rescaled data show that for sample numbers beyond 6, alignment thresholds are relatively invariant for both 1st- and 2nd-order stimuli. The presentation of fewer than 6 samples, in most cases, results in a reduction in alignment performance. A similar pattern of results was

![Figure 3](https://jov.arvojournals.org/pdfaccess.ashx?url=/data/journals/jov/933534/)
observed for both small (1.9 arcmin) and large (7.7 arcmin) sample separations. Finally the influence of sample separation was examined using a fixed standard deviation (0.333°) target Gaussian. It was necessary to ensure that the number of samples present in the stimulus was always above four-six samples, as the earlier results have shown that presenting stimuli with fewer than this number of samples causes thresholds to rise sharply. The sample separations employed were 0, 1.926, 3.21, 4.494, 6.42, 7.704, and 8.988 arcmin with the corresponding number of samples present being 255, 64, 42, 32, 23, 19, and 17. All other procedural details were the same as in the previous experiments.

Figure 4. Alignment thresholds plotted as a function of sample number for each sample separation, for 1st-order (a and b) and 2nd-order (c and d) stimuli. Data are derived from a re-scaling process that eliminates threshold elevations resulting from changes in the width of the underlying Gaussian distribution. Error bars represent standard deviations and their derivation from the Gaussian width data (Figures 2a and 2b), and the threshold measurements for varying sample numbers (Figures 3a–3d) is described in the Appendix A.

Figure 5. Alignment thresholds plotted as a function of sample separation for three observers. Solid lines and filled symbols represent performance with 1st-order stimuli while dashed lines and open symbols represent performance with 2nd-order stimuli. The standard deviation of the target Gaussian was fixed at 0.333°.
Discussion

The results of the present study clearly show that the critical variable determining alignment performance is the number of available samples rather than sample separation—as proposed by Morgan and Watt (1982). Instead, our findings are consistent with Kontsevich and Tyler (1998) who showed that positional judgements were invariant with sample separation for a fixed distribution width provided at least 3–4 samples were present. Kontsevich and Tyler stated that these alternate accounts could be explained by differences in stimulus arrangement. In Morgan and Watt’s task, observers were required to make a relative localization judgment of two sampled difference of Gaussian functions (DoG), one presented above fixation and the other below. Importantly, the sample positions above and below fixation were identical, regardless of the sample separation value. Kontsevich and Tyler suggested that the similarity between the two distributions to be localized may have allowed a low level or local luminance comparison to mediate judgements. However, our results make an explanation based on the local similarity of the two distributions seem unlikely. In the present study, identical distributions were presented above and below fixation in a similar fashion to Morgan and Watt. The critical difference between our study and theirs was that sample location was not fixed but randomly jittered from trial to trial. This process, coupled with the use of a dynamic noise carrier, ensured that any local luminance cue, if present, would be rendered unreliable and instead judgements would need to be based on the distribution as a whole. A comparison of localization thresholds suggests that this may indeed be the case. Morgan and Watt reported threshold values in the region of 10–20 arc seconds in comparison with our optimum thresholds that are between 1–2 arc minutes for luminance-defined distributions (at a sample separation of 1.9 arcmin). The thresholds found in this study are more similar to those reported by Kontsevich and Tyler (1998). Their slightly poorer threshold values (around 4–5 arcmin) probably results from the fact that their luminance-defined test and reference stimuli were of opposite polarity—a factor known to elevate thresholds on positional tasks (Levi, Jiang, & Klein, 1990; Levi & Westheimer, 1987; O’Shea & Mitchell, 1990). The small differences in absolute sample number between this study (6 samples) and Kontsevich and Tyler (3–4 samples) likely reflects the fact that we sampled our distributions both above and below fixation rather than localizing a single sampled distribution relative to a fixed luminance reference marker. Interpolation performance for 2nd-order, or contrast-defined, distributions were qualitatively similar to their 1st-order counterparts. Threshold values were generally higher for localizing 2nd-order distributions but the overall pattern of results for both sample separations was identical. Once again, if 6 or more samples were available to the observer, thresholds became invariant of sample number. Sukumar and Waugh (2007) have previously shown larger summation areas for 2nd-order stimuli when mapping thresholds against stimulus area while employing stimuli created using very similar methods. Given that result it would be expected that 2nd-order stimuli should be able to tolerate broader sample separations before performance declined. Figure 5 presents performance for three of the observers as a function of sample separation. While performance does decline with increasing separation there is no apparent difference in dependence on separation between the results for 1st- and 2nd-order stimuli. Sukumar and Waugh (2007) estimated that the summation zones for the detection thresholds of 1st- and 2nd-order stimuli presented at the fovea were fitted by Gaussians with standard deviations (σ) of approximately 13 and 37 arcmin respectively. These values seem unlikely to pertain to our localization task, since at our largest sample separation of 8.7 arcmin only two or three samples would fall into a ±1σ summation zone for 1st-order stimuli while 2nd-order detectors should be able to use approximately 8. This should result in a larger decline in performance for 1st- than 2nd-order stimuli at the largest sample separation. That result is not obtained here. However, estimates of the size of Ricco’s area, the region in which stimulus intensity and area are inversely proportional at threshold, do vary across observers and with eccentricity (Sukumar & Waugh, 2007), even over the relatively small extent of our current stimuli. The area also varies with luminance (Cornsweet & Yellott, 1985) although Sukumar and Waugh (2007) used a mean luminance of 52 cd.m⁻² (Waugh, 2008, personal communication) which is very similar to the 54.2 cd.m⁻² used here. In future the summation zones for contrast detection thresholds and localization should be determined within the same observers and using the same stimuli.

Previous work has shown that perceived position is markedly altered by prior adaptation (Whitaker et al., 1997). However, these distortions of positional representation are confined to situations where the adapting and test pattern overlap in their visual composition. For example, adapting and testing with either 1st-order or 2nd-order stimuli produces large shifts in perceived position, but adapting with 1st-order and testing with 2nd-order has little or no effect. This tells us that positional judgements for these two types of visual stimuli are ostensibly encoded independently of each other. Despite this, the cortical mechanisms responsible for reconstructing and localizing a distribution from a limited set of samples show remarkable overlap in their operational characteristics. Given that the spatial interpolation mechanisms and feature extraction stage (Whitaker & McGraw, 1998) are similar for both types of stimuli, the elevated thresholds for localizing 2nd-order stimuli may...
arise for other reasons. Several possibilities warrant future investigation. First, we did not precisely match the relative detectability of the stimuli but instead chose a high, easily detected contrast (Gaussian contrast of 0.5 in both Equations 1 and 2). The poorer performance may also arise from greater noise sensitivity, either at the initial filtering stage where the 2nd-order detectors employ filters tuned to higher spatial frequencies and are thus better able to resolve the noise, or at the signal demodulation stage of the filter-rectify-filter cascade. This study did not attempt to determine the causes of the difference in overall level of performance but instead focused on the role of sampling of the underlying distribution.

A more recent study conducted by Likova and Tyler (2003), proposed a conceptual framework for spatial interpolation based on stereoscopic depth. Their subjects reported that the sampled luminance distribution evoked a strong perception of stereoscopic depth with the brightest samples appearing closer in depth. To show that luminance and disparity information are fed into a unitary map they created sampled stimuli that consisted of both luminance and disparity information. They found that the disparity information could be used to null the perceived depth evoked by the luminance samples and that as soon as the luminance surface appeared flat the underlying distribution became impossible to localize. They concluded from this that thresholds ultimately depend upon interpolation of the three-dimensional profile of the underlying distribution. If spatial interpolation is driven by a mechanism operating at the level of a generic depth map then a perceived disparity should also be evident and mutable (with opposite signed disparity), in our sampled 2nd-order stimuli. Furthermore, disparity discrimination thresholds are considerably poorer for 2nd-order stimuli as compared with equivalent 1st-order stimuli of the same physical contrast (Sato & Nishida, 1993). Recent evidence also suggests that 2nd-order mechanisms do not contribute to the perception of three-dimensional surface shape (Hess, Ledgeway, & Dakin, 2000; Ziegler & Hess, 1999). Given this information, it is difficult to see how the reconstruction of a sparse 2nd-dimensional surface shape (Hess, Ledgeway, & Dakin, 1993). Recent evidence also suggests that 2nd-order mechanisms do not contribute to the perception of three-dimensional surface shape (Hess, Ledgeway, & Dakin, 2000; Ziegler & Hess, 1999). Given this information, it is difficult to see how the reconstruction of a sparse 2nd-dimensional surface shape (Hess, Ledgeway, & Dakin, 1993). Recent evidence also suggests that 2nd-order mechanisms do not contribute to the perception of three-dimensional surface shape (Hess, Ledgeway, & Dakin, 2000; Ziegler & Hess, 1999). Given this information, it is difficult to see how the reconstruction of a sparse 2nd-dimensional surface shape (Hess, Ledgeway, & Dakin, 1993). Recent evidence also suggests that 2nd-order mechanisms do not contribute to the perception of three-dimensional surface shape (Hess, Ledgeway, & Dakin, 2000; Ziegler & Hess, 1999). Given this information, it is difficult to see how the reconstruction of a sparse 2nd-dimensional surface shape (Hess, Ledgeway, & Dakin, 1993). Recent evidence also suggests that 2nd-order mechanisms do not contribute to the perception of three-dimensional surface shape (Hess, Ledgeway, & Dakin, 2000; Ziegler & Hess, 1999). Given this information, it is difficult to see how the reconstruction of a sparse 2nd-dimensional surface shape (Hess, Ledgeway, & Dakin, 1993). Recent evidence also suggests that 2nd-order mechanisms do not contribute to the perception of three-dimensional surface shape (Hess, Ledgeway, & Dakin, 2000; Ziegler & Hess, 1999). Given this information, it is difficult to see how the reconstruction of a sparse 2nd-dimensional surface shape (Hess, Ledgeway, & Dakin, 1993). Recent evidence also suggests that 2nd-order mechanisms do not contribute to the perception of three-dimensional surface shape (Hess, Ledgeway, & Dakin, 2000; Ziegler & Hess, 1999). Given this information, it is difficult to see how the reconstruction of a sparse 2nd-dimensional surface shape (Hess, Ledgeway, & Dakin, 1993). Recent evidence also suggests that 2nd-order mechanisms do not contribute to the perception of three-dimensional surface shape (Hess, Ledgeway, & Dakin, 2000; Ziegler & Hess, 1999). Given this information, it is difficult to see how the reconstruction of a sparse 2nd-dimensional surface shape (Hess, Ledgeway, & Dakin, 1993). Recent evidence also suggests that 2nd-order mechanisms do not contribute to the perception of three-dimensional surface shape (Hess, Ledgeway, & Dakin, 2000; Ziegler & Hess, 1999). Given this information, it is difficult to see how the reconstruction of a sparse 2nd-dimensional surface shape. For the data in Figure 3, the width of the sampled Gaussian increases as the number of samples increase. So the thresholds are, in effect, a function of both sample number and Gaussian width, i.e.: Th(Sn, Gw). In order to see how just sample number affects threshold, the thresholds from Figure 3, Th(Sn, Gw), were re-scaled by normalizing with the threshold data from Figure 2. Thus Th(Gw) was calculated from the linear regressions of the Figure 2 data for the widths of each data point in Figure 3. These values were then used to re-scale the data in Figure 3 as follows

\[
\langle \text{Th} \rangle = \frac{\text{Th(Sn,Gw)}}{\text{Th(Gw)}},
\]

and these are the threshold values shown in Figure 4.

The data in Figure 2 shows how threshold is affected by unsampled Gaussian width. For each subject, a linear regression was performed to obtain the value for the gradient (m) and y-intercept (k) for the line of best fit. This gives an equation for threshold as a function of Gaussian width for each subject

\[
\text{Th(Gw)} = m \times \text{Gw} + k.
\]

For the data in Figure 3, the width of the sampled Gaussian increases as the number of samples increase. So the thresholds are, in effect, a function of both sample number and Gaussian width, i.e.: Th(Sn, Gw).

The propagation of errors from the re-scaling was calculated in the following way (Goodman, 1960). For the linear regression, the errors in the slope and y-intercept will be correlated and so the propagated error depends also on the correlation coefficient, \(r_{mk}\). The variance of the linear regression thresholds, Th(Gw), is

\[
\text{var(Th(Gw))} = \text{var(m)} \cdot \text{Gw}^2 + \text{var(k)} + 2 \times \text{Gw} \cdot r_{mk} \cdot \text{sqrt} \left( \text{var(m)} \cdot \text{var(k)} \right).
\]

Using the empirical variance for each threshold data point, Th(Sn, Gw), in Figure 3, the variances of the re-scaled thresholds were calculated as

\[
\text{var(Th)} = \langle \text{Th} \rangle^2 \cdot \text{var(Th(Sn,Gw))}/\text{Th}^2(\text{Sn,Gw}) + \text{var(Th(Gw))}/\text{Th}^2(\text{Gw}),
\]

and the error bars in Figure 4 correspond to the square root of this variance, i.e.: the standard deviation of \(\langle \text{Th} \rangle\).

## Appendix A

### Threshold re-scaling calculation for data in Figure 4

The data in Figure 2 shows how threshold is affected by unsampled Gaussian width. For each subject, a linear regression was performed to obtain the value for the gradient (m) and y-intercept (k) for the line of best fit. This gives an equation for threshold as a function of Gaussian width for each subject

\[
\text{Th(Gw)} = m \times \text{Gw} + k.
\]

For the data in Figure 3, the width of the sampled Gaussian increases as the number of samples increase. So the thresholds are, in effect, a function of both sample number and Gaussian width, i.e.: Th(Sn, Gw).

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\[
\text{var(Th(Gw))} = \text{var(m)} \cdot \text{Gw}^2 + \text{var(k)} + 2 \times \text{Gw} \cdot r_{mk} \cdot \text{sqrt} \left( \text{var(m)} \cdot \text{var(k)} \right).
\]

Using the empirical variance for each threshold data point, Th(Sn, Gw), in Figure 3, the variances of the re-scaled thresholds were calculated as

\[
\text{var(Th)} = \langle \text{Th} \rangle^2 \cdot \text{var(Th(Sn,Gw))}/\text{Th}^2(\text{Sn,Gw}) + \text{var(Th(Gw))}/\text{Th}^2(\text{Gw}),
\]

and the error bars in Figure 4 correspond to the square root of this variance, i.e.: the standard deviation of \(\langle \text{Th} \rangle\).
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