Computation of slant specified by cues

Slant from perspective convergence, part 1:
Construction of the projection \((x1, y1)\) on the screen (blue) of point \((x0, y0)\) of the rectangle (red) that is slanted by \(\phi\) about the vertical axis.
\(zc\) is the center of projection.

\[
y1 = -zc \cdot y0 / (x0 \cdot \sin[\phi] - zc);
\]
\[
x1 = -zc \cdot x0 \cdot \cos[\phi] / (x0 \cdot \sin[\phi] - zc);
\]
Slant from perspective convergence, part 2:
Construction of point \((x_2, z_2)\) lying on the plane (red) defined by the unchanged height \((y_0)\) of the rectangle after that the screen has been slanted by \(\sigma\).

\(z_v\) is the viewing point.

\[
\begin{align*}
zp &= x_1 \cdot \sin[\sigma] - z_v; \\
(* \ zp \ is \ the \ z-distance \ from \ the \ point \ on \ screen \ to \ zv *)
xp &= x_1 \cdot \cos[\sigma]; (* \ xp \ is \ the \ distance \ from \ the \ point \ on \ screen \ to \ the \ z-axis *)
zpp &= zp \cdot y_0 / y_1; (* \ zpp \ is \ the \ z-distance \ from \ z2 \ to \ zv *)
z2 &= zpp + zv;
x2 &= xp \cdot zpp / zp;
upsilon &= \text{ArcTan}[(z2) / x2];
\text{Simplify}[\upsilon] / \cdot zv \rightarrow zc \cdot r (* \ r \ is \ the \ ratio \ of \ zv \ to \ zc *)
\text{ArcTan}[\text{Tan}[\sigma] + r \ \text{Sec}[\sigma] \ \text{Tan}[\phi]]
\end{align*}
\]
The viewing point coincides with the center of projection: $z_v = z_c$

$$\upsilon_1 = \text{Simplify}[\upsilon] / . z_v / z_c \rightarrow 1$$

Plot3D[$\upsilon_1$, \{\(\sigma\), \(-\frac{\pi}{2}\), \(\frac{\pi}{2}\)\}, \{\(\phi\), \(-\frac{\pi}{2}\), \(\frac{\pi}{2}\)\},
BaseStyle -> {21, FontFamily -> "Helvetica"},
ColorFunction -> Function[\{\(\sigma\), \(\phi\), \(\upsilon_1\)\},
ColorData["RedBlueTones", "Reverse"]][\(\phi\)],
BoxRatios -> \{1, 1, 1\}]

ArcTan[Tan[\(\sigma\)] + Sec[\(\sigma\)] Tan[\(\phi\)]]
The viewing point is further away than the center of projection: \( z_v \gg z_c \).

\[
\upsilon_1a = \text{Simplify}[\upsilon]/. z_v/z_c \rightarrow 10
\]

\[
\text{Plot3D}[\upsilon_1a, \{\sigma, -\pi/2, \pi/2\}, \{\phi, -\pi/2, \pi/2\},
\text{BoxRatios} \rightarrow \{1, 1, 1\}]
\]

\[
\text{ArcTan}[\text{Tan}[\sigma] + 10 \text{Sec}[\sigma] \text{Tan}[\phi]]
\]
The viewing point is closer than the center of projection: \( z_v \ll z_c \)

\[
\text{upsilon1b} = \text{Simplify}[\text{upsilon}] / z_v / z_c \rightarrow 0.1
\]

Plot3D[upsilon1b, \{\(\sigma\), \(-\Pi/2\), \(\Pi/2\)\}, \{\(\phi\), \(-\Pi/2\), \(\Pi/2\)\},
BaseStyle \rightarrow \{21, \text{FontFamily} \rightarrow "Helvetica"\},
ColorFunction \rightarrow \text{Function}[\{\sigma, \phi, \text{upsilon1}\},
\text{ColorData}["RedBlueTones", "Reverse"]][\phi]],
BoxRatios \rightarrow \{1, 1, 1\}

\[\text{ArcTan}[\text{Tan}[\sigma] + 0.1 \sec[\sigma] \text{Tan}[\phi]]\]
Relationship between line inclination ($\alpha$), line separation ($\beta$) and slant from perspective convergence

\[ \beta = .; \ zv = .; \ xp = .; \]
\[ p1 = \{xp, zv \tan[\beta] + xp \tan[\alpha], 0\}; \]
\[ (* \ point \ of \ a \ line \ at \ height \ \beta \ on \ screen \ inclined \ by \ \alpha *) \]
\[ p2 = \{0, 0, zv\}; \]
\[ (* \ viewing \ point *) \]
\[ p3 = \{x, y, z\}; \]
\[ (* \ virtual \ point \ that \ is \ associated \ with \ p1 *) \]
\[ c1 = (1 - t) p2 + t p1; \]
\[ (* \ p3 \ on \ the \ line \ between \ p1 \ and \ p2 *) \]
\[ c2 = zv \tan[\beta]; \]
\[ (* \ pictorial \ line \ is \ horizontal *) \]
\[ e = \text{Eliminate}[p3 = c1 \&\& p3 = \{x, c2, z\}, \{t, y\}]; \]
\[ s = \text{Solve}[e, \{x, z\}]; \]
\[ \upsilon1c = \text{ArcTan}[z \div x]. \]
\[ \beta = 6 \text{ Degree}; \]
\[ \text{Plot}[\upsilon1c, \{\alpha, -\pi / 2, \pi / 2\}, \]
\[ \text{PlotRange} \to \{\{-\pi / 5, \pi / 5\}, \{-\pi / 2, \pi / 2\}\}, \]
\[ \text{PlotStyle} \to \{\text{Black, Thick}\}, \]
\[ \text{AspectRatio} \to 1 / \text{GoldenRatio}, \]
\[ \text{Ticks} \to \{\{-4 \beta, "-4\beta"\}, \{-2 \beta, "-2\beta"\}, \{2 \beta, "2\beta"\}, \{4 \beta, "4\beta"\}\}, \]
\[ \{\{-\pi / 2, \pi / 2\}\}, \]
\[ \text{BaseStyle} \to \{21, \text{FontFamily} \to "Helvetica"\}]; \]

\{\text{ArcTan}[\cot[\beta] \tan[\alpha]]\}

\[ \frac{\pi}{2} \]
\[ -4\beta \quad -2\beta \quad 2\beta \quad 4\beta \]
\[ -\frac{\pi}{2} \]
Slant from horizontal compression alone

\[
\begin{align*}
z v & = \cdot; \\
\upsilon_2^p & = \text{ArcCos}[x_2 / x_0] \\
\upsilon_2^m & = -\text{ArcCos}[x_2 / x_0] \\
\text{Plot3D}[(\upsilon_2^p, \upsilon_2^m), \{\sigma, -\pi / 2, \pi / 2\}, \\
\{\phi, -\pi / 2, \pi / 2\}, \text{BaseStyle} \rightarrow \{21, \text{FontFamily} \rightarrow "Helvetica"\}, \text{ColorFunction} \rightarrow \text{Function}[\{\sigma, \phi, \upsilon_2^p\}, \\
\text{ColorData}["RedBlueTones", "Reverse"]][\phi]], \\
\text{BoxRatios} & \rightarrow \{1, 1, 1\} \\
\text{ArcCos}[\cos[\sigma] \cos[\phi]] \\
-\text{ArcCos}[\cos[\sigma] \cos[\phi]]
\end{align*}
\]
Slant from horizontal compression disambiguated by the sign of perspective-specified slant

$$z v = . ;$$

$$\upsilon_3 = \text{Simplify}\left[\text{Sign}\left[\upsilon_1\right] \text{ArcCos}\left[\frac{x_2}{x_0}\right]\right]$$

Plot3D[$\upsilon_3$, \{\(\sigma\), \(-\pi/2\), \(\pi/2\)\}, \{\(\phi\), \(-\pi/2\), \(\pi/2\)\},
  BaseStyle -> {21, FontFamily -> "Helvetica"},
  ColorFunction -> Function[\{\(\sigma\), \(\phi\), \(\upsilon_3\)\},
    ColorData[{"RedBlueTones", "Reverse"}][\(\phi\)],
  BoxRatios -> \{1, 1, 1\}]

ArcCos[\(\cos[\sigma] \cos[\phi]\)] \text{Sign}[\text{ArcTan}\left[\text{Tan}[\sigma] + \text{Sec}[\sigma] \text{Tan}[\phi]\right]]$$
Slant from screen cues

\[ \upsilon_4 = \sigma \]

Plot3D[\[ \upsilon_4, \{\sigma, -\pi/2, \pi/2\}, \{\phi, -\pi/2, \pi/2\},
BaseStyle \to \{21, FontFamily \to "Helvetica"\},
ColorFunction \to Function[\{\sigma, \phi, \upsilon_4\},
ColorData["RedBlueTones", "Reverse"]][\phi],
BoxRatios \to \{1, 1, 1\}]