Effect of contiguity and figure-ground organization on the area rule of lightness

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In a simple two-dimensional (2D) display composed of two uniform surfaces with different luminances, the lightness of the darker surface varies as a function of its relative area while its luminance is held constant (Gilchrist & Radonjić, 2009; Li & Gilchrist, 1999). This phenomenon is known as the area rule of lightness, and although it is extensively studied in the literature, the underlying principles are still largely unknown. Here, using computer-generated stimuli, we investigated the effects of contiguity and figure-ground organization on the area rule of lightness. Stimuli were 2D disks composed of radial sectors with high (25 cd/m²) or low (8 cd/m²) luminance. On each trial, observers judged the lightness of the sectors by adjusting the luminance of a matching patch. Four conditions were tested. In the contiguous condition, there were one dark and one light sector; in the noncontiguous condition, both the light and dark surfaces were split into four equal radial sectors. Figure and ground conditions were generated by adding small contextual elements to the stimulus. We found that the area rule applied under all conditions; however, the functional form of the effect showed marked differences across conditions. Taken together, our results show that both high-level (e.g., perceptual grouping, figure-ground organization) and low-level (e.g., spatial-summation) mechanisms play a role in the area rule of lightness.

Introduction

The lightness of a surface depends not only on its luminance but also on its geometry and the context within which it is viewed (Boyaci, Doerschner, Snyder, & Maloney, 2006; Gilchrist, 2006; Kingdom, 2011; Maloney, Boyaci, & Doerschner, 2005; Maloney, Gerhard, Boyaci, & Doerschner, 2010). One of the geometric factors that affect the lightness of a surface is its perceived area (Daneyko, 2011). In a simple two-dimensional (2D) display composed of two uniform surfaces with different luminances, the lightness of the darker surface varies with its relative area while its luminance is held constant (Gilchrist & Radonjić, 2009; Li & Gilchrist, 1999; see Figure 1). Although the effect, dubbed as the “area rule of lightness,” is well established in the literature, the underlying principles are still not very well understood, and it is not clear how the area rule emerges as a result of high- and/or low-level visual processes (Gilchrist & Radonjić, 2009; Li & Gilchrist, 1999).

Here we seek to address several of the open questions about the area rule of lightness. First, could contiguity have an effect on the area rule of lightness? In other words, how would the rule be affected if the darker region were not a single contiguous surface but split into smaller parts? If the rule is the result of very early low-level processes such as spatial summation in early retinotopic brain areas (Angelucci & Shushruth, 2013; Richards, 1967), we would expect a measurable difference between contiguous and noncontiguous configurations. If, on the other hand, it is the result of higher-level processes, such as a process that perceptually combines separate sectors, then we might observe little or no difference between the two configurations. More specifically, we would expect the lightness to vary more closely with the area of individual sectors under the first model and with combined area of sectors under the second model. Second, could figure-ground organization play any role in the area rule of lightness?
role in the area rule of lightness? For example, is it possible that as the relative area of a surface increases it becomes more likely to be interpreted as ground, and therefore its lightness varies with its probability of being ground? Or could it be that the area rule applies only to surfaces that are interpreted as figure? Because the stimuli used in previous studies had no clear figure-ground organization cues, it is hard to guess if the visual system interpreted any of the surfaces as figure or ground. What if geometric cues were introduced that suggest that a given surface is figure or ground?

Previous studies in the literature did not directly investigate the relationship between area rule of lightness and figure-ground organization and contiguity (however see Radonjić and Gilchrist, 2014, where contiguity is studied using real scenes). Here, in a series of experiments, we address these previously unexplored questions and seek to shed light on the underlying high- and low-level mechanisms of the area rule of lightness.

Methods

Two series of experiments were conducted to investigate the effects of contiguity and figure-ground organization on the area rule of lightness. General methods are described in this section; methods specific to each experiment are described below.

Participants

Nine paid participants with normal or corrected-to-normal vision and no known visual or neurological disorders completed two experiments in randomized and counterbalanced order. Participants were graduate and undergraduate students and a faculty member working in the National Magnetic Resonance Research Center (UMRAM), Bilkent University. All were unaware of the hypotheses tested. Informed written consents were obtained prior to the start of the experiments in accordance with the rules and protocols approved by the Human Ethics Committee of Bilkent University.

Software and apparatus

Stimuli were presented to the participants using a computer-controlled system in an otherwise dark room. Participants were seated 75 cm from a 21-in. LCD screen (NEC 2190UXP) with their heads stabilized using a head and chin rest. The system was controlled by a desktop personal computer running on Fedora Linux (version 10), equipped with an NVidia graphics card (Quadro FX 1700). Gray-scale color look-up tables were prepared by directly measuring luminance values of uniform gray patches with a spectrophotometer (SpectroCAL, Cambridge Research Systems, Kent, UK). The tables were then used for displaying desired luminance values on the screen. The maximum luminance achievable was 265 cd/m² (Commission Internationale de l’Éclairage: \(x = 0.3314, y = 0.3505\); the chromaticity was approximately constant across all gray levels used in the experiment). Experimental software was written by us in the Java programming platform.

Stimuli and procedures

The stimuli were 2D images of disks composed of radial sectors with two different luminance values, which were 8 cd/m² and 25 cd/m². The global background of the rest of the display had a luminance of 2 cd/m². In Experiment 1, the dark and light sectors...
were either contiguous (“contiguous” condition, Figure 1) or they were split into four equal portions (“noncontiguous condition,” Figure 2, top right). The quadruple of sectors were distributed equidistantly on the disk in the noncontiguous condition. In Experiment 2, additional contextual elements were introduced, and the sectors appeared either as figure or as ground (Figure 4). The relative areas of the dark and light sectors tested were 0.12, 0.24, 0.38, 0.5, 0.62, 0.76, and 0.88 in Experiment 1 and 0.125, 0.25, 0.375, 0.5, 0.625, 0.75, and 0.875 in Experiment 2. The diameter of the disks was 10° of visual angle. The exact configuration was randomized at each trial by randomly varying the starting polar angle of the first dark sector.

In each trial, participants were asked to judge the lightness of the tested sector by adjusting the luminance of a matching patch placed over a random noise pattern at the lower part of the screen. The sector under test was indicated by the words Dark and Light displayed near the left border of the random noise background (Figure 1). The random noise background was 15° by 3° of visual angle (width × height), and the matching patch was 1.5° by 1.5° of visual angle. To generate the random noise background, the luminance of each pixel was drawn from a Gaussian distribution with minimum luminance of zero and maximum luminance of 265 cd/m², and then the resulting image was convolved with a 6 × 6 uniform filtering kernel. Each experiment was completed in two sessions. In a single session, every configuration was tested five times, for a total number of trials equal to 140 across the two sessions and 560 across four conditions.

Observers sometimes confused the target; for example, when instructed to perform the setting for light sector, they did for the dark sector. Participants informed us immediately when they noticed such an error. We also removed outliers, which obviously came from such erroneous trials, before further analyses (the number of such removed trials was 112 of 5,040 trials across all conditions and observers).

Results

Experiment 1: Test of contiguity

In this experiment, we tested the effect of contiguity on observers’ lightness judgments under two conditions. In the contiguous condition, stimuli were disks composed of contiguous sectors of dark and light regions (Figure 2, left panel). In the noncontiguous condition, the stimulus disks were composed of four equal-sized dark sectors distributed between four equal-sized light sectors (Figure 2, right panel).

Figure 2 shows the average settings across observers as a function of the relative total area of the dark sector. Under both the contiguous and noncontiguous conditions, we found a significant effect of relative area on the lightness judgments of the dark sector, \( F(6, 605) = 40.99, p < 0.001 \), for the contiguous and \( F(6, 623) = 54.66, p < 0.001 \), for the noncontiguous condition but not of the light sector, \( F(6, 603) = 0.99, p = 0.43 \), for the contiguous and \( F(6, 618) = 0.32, p = 0.93 \), for the noncontiguous condition. Overall average settings were higher in the contiguous condition for both the dark and light sectors, \( t(1,240) = 17.83, p < 0.001 \), for dark sectors and \( t(1,233) = 6.22, p < 0.001 \), for light sectors (Figure 3). For all relative areas, average settings for dark sectors were higher under the contiguous condition than under the noncontiguous condition (\( p < 0.001 \)), whereas for the light sector, contiguous condition settings were significantly higher than noncontiguous condition settings only for relative areas 0.38, 0.5, and 0.62 (\( p < 0.007 \), corrected for seven comparisons). These results are largely consistent with the previous literature and show that the lightness of the dark sector but not the light sector varies with relative area (Gilchrist & Radonjić, 2009; Li & Gilchrist, 1999).

Functional form of the area rule under contiguous and noncontiguous conditions

To fully investigate the functional form of the area rule under the two contiguity conditions, we performed further model comparisons. Namely, we compared the following models:

Model – B (Linear Model): \( L = a + b \times A \)

Model – C (2 – Phase Model): \( L = \begin{cases} a & \text{for } A \leq 0.5 \\ a + b \times (A - 0.5) & \text{for } A > 0.5 \end{cases} \)

Model – D (Quadratic Model): \( L = a + b \times A + c \times A^2 \);

where \( L \) is the observer’s setting (in units of tested luminance), \( A \) is the relative area, and \( a, b, \) and \( c \) are parameters to estimate. To compare Model B and Model D, we employed a parametric bootstrap procedure as outlined in Appendix A (Efron & Tibshirani, 1993; Kingdom & Prins, 2009). Because Model C is not nested under Model B or Model D, we employed the Bayesian information criterion (BIC) approach for their comparisons: \( \text{BIC}_m = -2 \times \log(L_m) + \log(n) \times K_m \), where \( L_m \) is the likelihood of model \( m \) using maximum likelihood estimates of its parameters, \( n \) is the number of observations, and \( K_m \) is the number of free parameters in the model. Models with smaller BIC values are preferred (Schwarz, 1978).

Contiguous condition: Model C (two-phase model) is not preferred over either Model B (linear model) or...
Comparing Model B and Model D, we could not reject the null hypothesis that the experimental sample is generated by Model B at the achieved significance level (ASL) of 0.05. Thus, our results suggest that a linear model describes the data best (Figure 3).

These results are not in complete agreement with some studies in the literature, which suggest two-phase dependence on area (Gilchrist & Radonjić, 2009; Li & Gilchrist, 1999). To further investigate the validity of our results, we conducted another experiment in which we directly compared lightness estimates of dark sectors with relative areas of 0.125, 0.25, and 0.375 in a two-alternative forced-choice procedure. If lightness estimates for relative areas less than 0.5 did not depend on relative area as suggested by the two-phase model, we would expect to find no difference in lightness between them. However, the results were in line with the findings reported above. There was a difference between lightness estimates of the sectors with different relative areas less than 0.5. See Appendix B for details of this experiment.

Noncontiguous condition: We found that Model C (two-phase model) is preferred over Model B (linear model) ($\Delta \text{BIC}_{B,C} = 19.48$) but not over Model D (quadratic model; $\Delta \text{BIC}_{D,C} = -3.30$). A comparison of Model B and Model D showed that Model D provides a statistically better fit (ASL $< 0.001$). These analyses show that for the noncontiguous condition, a quadratic model is preferred over other models (Figure 3). These results...
suggest that a high-level process that perfectly combines separate sectors cannot alone explain the findings.

Next, we investigated whether the lightness varies with the relative area of individual sectors in the noncontiguous condition. What would we expect if the area rule was the result of a purely low-level mechanism? For example, because of excitatory horizontal interactions between cells that signal surface luminance ("luxotonic" cells) in retinotopic visual areas (Callaway, 2013; Penacchio, Otazu, & Dempere-Marco, 2013; Peters, Jans, van de Ven, De Weerd, & Goebel, 2010), the average neuronal response amplitude could increase. This kind of spatial summation mechanisms in early visual areas could lead to an increased perceived lightness for larger contiguous surfaces (Angelucci & Shushruth, 2013; Richards, 1967). If this were the only mechanism underlying the area rule of lightness, we would expect to find the effect dependent not on the total relative area of all sectors but on the relative area of individual sectors. To test this possibility, we performed new model comparisons. Namely, in the null hypothesis, lightness depends on the relative area of an individual sector (not combined surface area, "pure low-level" model). In this hypothesis, the relation between relative area and lightness is given by the best model found in the contiguous condition (up to an additional constant factor). Alternative possibilities included all the models given in Equation 1 with parameters free to vary. We rejected the null hypothesis (ASL < 0.001), meaning that a purely low-level account of the area rule is not supported by the data.

Experiment 2: Effect of figure-ground organization

We next investigated whether figure-ground organization has any effect on the area rule. For this purpose, we used a set of stimuli similar to those used in Experiment 1, the noncontiguous condition. However, by adding contextual elements, we rendered the sectors perceived as either ground or figure ("ground" and "figure" conditions, referring to the dark sectors, Figure 4; Fang, Boyaci, & Kersten, 2009).

The average settings are plotted in Figure 4 for dark sectors and in Figure 5 for light sectors. For both the figure and ground conditions, we found a significant effect of area on the lightness of the dark sector, $F(6, 620) = 7.3253, p < 0.001$, for the ground condition and $F(6, 616) = 19.991, p < 0.001$, for the figure condition but not of the light sector, $F(6, 618) = 1.3225, p = 0.245$, for the figure condition and $F(6, 613) = 0.4244, p = 0.863$, for the ground condition. Overall average settings were higher in the ground condition for the dark sectors, $t(1,248) = 13.06, p < 0.001$. Moreover, at each relative area, the average setting for the dark
sectors was higher in the ground condition than in the figure condition ($p < 0.001$ for all areas, corrected for multiple comparisons). For the light sectors, the overall average of settings was slightly higher in the figure condition, $t(1,243) = 1.7, p = 0.044$, and there was no significant difference between ground and figure conditions for any of the relative areas at a significance level of 0.05 (corrected for multiple comparisons).

**Functional form of the area rule under figure and ground conditions**

As in Experiment 1, we compared how well linear, two-phase, and quadratic models fit the data using Equation 1.  

**Ground condition:** Model C (two-phase model) in Equation 1 is preferred over both Model B (linear model) and Model D (quadratic model; $\Delta BIC_{B,C} = 14.13, \Delta BIC_{D,C} = 4.45$). These results suggest that in the ground condition, the area rule is applicable only if the total relative area of the dark sectors is larger than 0.5 (Figure 4).  

**Figure condition:** Our analyses show that Model C (two-phase model) is not preferred over Model B (linear model) and Model D (quadratic model; $\Delta BIC_{B,C} = -2.65, \Delta BIC_{D,C} = -2.85$). Parametric bootstrap analysis shows that a quadratic model provides the best fit to the data ($ASL < 0.05$; Figure 4).  

Given that the data were best described with the quadratic model under both the noncontiguous condition of Experiment 1 and the figure condition of Experiment 2, and the geometric similarity of the stimuli in those two configurations, we next tested how consistent the patterns of results are across these two conditions. Namely, we compared how well the two data sets can be modeled with the same parameters in the quadratic model, up to an additive constant. If figure-ground organization plays no role in the area rule, we would expect little or no difference between the two conditions. Results show that a single model fit is not as good as two separate fits at an ASL of 0.05. The fit for figure condition in Experiment 2 was slightly flatter than the fit for the noncontiguous condition in Experiment 1 (Figure 6). Moreover, it is clearly seen in Figure 6 that the average settings are higher under the figure condition. These results suggest that figure-ground organization has an effect on the area rule of lightness investigated here.

**Discussion**

Under all configurations tested, we found that lightness of the dark sectors, but not light sectors, varied with their area while their luminance was kept constant. This main finding is consistent with the literature (Gilchrist & Radonjić, 2009; Li & Gilchrist, 1999) and shows that the area rule is applicable in computer-rendered scenes as well as in real scenes. Beyond this main finding, we observed marked differences in the results across different conditions.  

**Effect of contiguity**

We found that the area rule applies under both contiguity conditions; however, the patterns of results are different between the two conditions. First, on average, the dark sectors are judged lighter in the contiguous
Second, the area rule has different functional forms under the two conditions. For the contiguous condition, a linear model describes the data best, whereas a quadratic model fit is better for the noncontiguous condition. If in the noncontiguous condition the sectors were combined at a higher level in the visual system through perceptual organization, and the area rule was based on this combined area, we would expect no difference between the two conditions. But this was not the case, speaking against such a perfect perceptual organization explanation. Besides, the finding that on average, the dark sectors are judged lighter under the contiguous condition further highlights the role of low-level spatial summation mechanisms. On the other extreme, we tested whether the rule depended on the surface area of individual sectors in the noncontiguous condition. In this case, if the effect were purely low level, such as spatial summation in early retinotopic visual areas, than we would expect it to depend on the relative area of individual sectors. This analysis revealed that a purely low-level account was not able to explain the results either. The empirical effect was larger than predicted by the area of the individual sectors and smaller than predicted by the combined area of all sectors. In other words, lightness varies with the combined area of individual sectors, but the combination is not perfect. Taken together, these results suggest that there is an effect of contiguity and that the area rule of lightness relies on both high- and low-level mechanisms.

As our study was under preparation for publication, Radonjić and Gilchrist (2014) reported a study in which they investigated the effect of contiguity in real scenes. As in their earlier studies (Gilchrist & Radonjić, 2009), they used domes that cover the entire visual field of observers to test the area rule. Contrary to our findings, they report a complete breakdown of the area rule with noncontiguous configuration under these conditions (Radonjić & Gilchrist, 2014). The disagreement between our studies is likely to be because of differences between the stimuli. Whereas their stimuli (Radonjić & Gilchrist, 2014) were real and as simple as possible, ours were computer generated and relatively more complex. In our study, observers viewed not only the surfaces under investigation but also edges around the disk, the matching patch, the random noise background, and the larger context of the darkened laboratory. Nevertheless, the discrepancy between the results of Radonjić and Gilchrist (2014) and those presented here leads to further questions (see below).

**Effect of figure-ground organization**

We found that the area rule of lightness applies in both figure and ground conditions. However, there were marked differences between them. First, darker regions in the ground condition appeared lighter than those in the figure condition for equal surface areas, which is similar to what is observed in the Wolff effect shown in Figure 7 (Gilchrist, 2006). But we also found that the functional form of the rule is different in the figure and ground conditions: Whereas in the ground condition the area rule had a two-phase form (Equation 1), in the figure condition it had a quadratic form. If mechanisms underlying the rule were exclusively low level, we would expect no difference between figure and ground conditions. The differences we found under the two conditions support the hypothesis that high-level processes play a role in area rule of lightness.
When we compared the noncontiguous condition in Experiment 1 and figure condition in Experiment 2, we found that they had similar although not identical functional forms. This suggests that in the noncontiguous condition (Experiment 1), the dark sectors are interpreted more as figure. Together with the effects of contiguity found in Experiment 1, we conclude that the area rule is the result of combined effects of high- and low-level processes.

Is the rule two-phased?

In the literature, there is a debate concerning the exact functional form of the area rule of lightness in simple 2D configurations where dark and light sectors form contiguous surfaces. As noted in Gilchrist and Radonjić (2009) and Li and Gilchrist (1999), the dependence may have two phases in such configurations. In the first phase, for relative areas less than 0.5, there is no effect of relative area on lightness; in the second phase, for relative areas greater than 0.5, lightness varies with relative area. However, we could not find any evidence to support this in our results. The discrepancy with the literature could be because of the differences in experimental conditions, including real versus computer-generated scenes. Most importantly, previous studies (Gilchrist & Radonjić, 2009; Li & Gilchrist, 1999) were done with stimuli that filled the entire visual field of the participants. In our experiments, participants were exposed to the stimulus on the screen as well as a black global background of the computer screen and the darkened laboratory. It is interesting to note that only in the contiguous condition did we find that the linear model fit the data best. In the noncontiguous and figure conditions, the quadratic model was preferred over the two-phase model, and the two-phase model was preferred over the linear model. For the ground condition, we found that the two-phase model provided the best fit, whereas the quadratic model provided the second best fit. The fact that in the ground condition of Experiment 2 we found that the two-phase model described the data best raises the possibility that in the settings of the study by Gilchrist and Radonjić (2009), the dark sectors could have been perceived as ground. However, note that a quadratic model was not tested in previous studies (Gilchrist & Radonjić, 2009). It would be interesting to perform model comparisons on those data as well.

Open questions and challenges for lightness models

With the available evidence, we conclude that both low-level and high-level mechanisms underlie the area rule. But there are still many unanswered questions. It is still unclear how and why the lightness of the darker surface, and only of the darker surface, varies with its relative area. We conjecture that as the area of the sectors increases, the spatial summation over a larger area leads to an increased neuronal activity, and this leads to an increased lightness judgment. Moreover, because of perceptual grouping, a summation occurs across disjoint regions. Our results also suggest that the rule follows different patterns under figure and ground conditions. It is not readily obvious why this is the case. It is also not straightforward to reconcile our findings with the results of earlier studies (Gilchrist & Radonjić, 2009; Li & Gilchrist, 1999). Whether the luminance of the global background of the computer screen (for example, black versus white) and belongingness to the background have any effect are other open questions (Daneyko, 2011).

It is not easy to reconcile our results with edge-integration models of lightness including the most recent improved version (Rudd, 2013), in which only the edges between the common global background and target regions are involved in lightness computations, not the edges between targets. Moreover, the model does not have a mechanism to take depth order information into consideration and therefore would not predict the pattern of results obtained in Experiment 2. Our results present both a challenge and opportunity to refine the edge-integration models.

According to the anchoring theory of lightness, the highest luminance area is perceived as white and the lightnesses of other surfaces are determined in relation to it (Gilchrist et al., 1999). Therefore, the model correctly predicts that the lightness of the light sectors does not vary with area. Based on only this fundamental principle, the area dependence of the lightness of the darker region cannot be predicted. However, this empirical observation is firmly built in to the theory (Gilchrist & Radonjić, 2009). In other words, anchoring theory has two rules. First, the highest luminance surface is perceived white and the lightnesses of other surfaces are determined in relation to it (Gilchrist et al., 1999). Therefore, the model correctly predicts that the lightness of the light sectors does not vary with area. Based on only this fundamental principle, the area dependence of the lightness of the darker region cannot be predicted. However, this empirical observation is firmly built in to the theory (Gilchrist & Radonjić, 2009). In other words, anchoring theory has two rules. First, the highest luminance surface is perceived white; second, the lightness of the darker region varies with its relative area. In this form, the theory could in principle predict, at least partly, the pattern of results in Experiment 2: The region that is perceived as background is amodally completed, and its total area is represented larger in the visual system (as in Wolff’s effect; Figure 7). Therefore, the dark region is perceived lighter in the ground condition than in the figure condition. However, with the current findings, it is not possible to explain why the dependence has two phases under the ground condition. It may either be that when the dark region is in the background, the area rule does not apply for relative areas below 0.5 or that the rule actually applies but the phenomenal area used by the visual system follows a nonlinear functional relationship with the retinal area.
Conclusions

Our main conclusion is that the area rule applies in all configurations tested in this study. Contrary to the findings in the literature, we could not find evidence supporting a two-phased dependence in simple 2D contiguous configurations. Taken together, our results suggest that the area rule of lightness emerges as a result of low- and high-level visual mechanisms. However, many questions still remain unresolved.

Keywords: lightness, brightness, area rule of lightness, perceptual organization, luminance

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Footnote

1 When the surface is perceived as background, it may be perceived as farther away from the observer. Surfaces that are perceived as farther away are judged to be lighter (Richards, 1967).

References


Penacchio, O., Otazu, X., & Dempere-Marco, L.
To compare which nested model fits the data better, we performed parametric bootstrap analyses (Efron & Tibshirani, 1993; F. Kingdom & Prins, 2009). Here, we outline the general approach. Suppose that there are two models to compare, Model L and Model F. Also suppose that $\Theta_L = \{\theta_1, \theta_2, \ldots, \theta_n\}$, $\Theta_F = \{\theta_1, \theta_2, \ldots, \theta_n, \theta_{n+1}\}$ are the free parameters in the two models. Because Model F has more free parameters, we call it the fuller model and Model L the lesser model. We first find the maximum likelihood estimates of parameters in both models and compute the maximum likelihood ratios:

$$K_{eL} = \frac{ML_{L}}{ML_{F}},$$

Next, we simulate the data using the parameters of the lesser model and then fit this simulated data using both the fuller and lesser models. We compute

$$K_{sL} = \frac{ML_{sL}}{ML_{sF}}.$$ We repeat this simulation for $B = 1,000$ times. The achieved significance level (ASL) is computed as the frequency of obtaining the data if the lesser model were true using the number of simulations in which $A_{L,F} < A_{L,F} : ASL = \#\{A_{L,F} < A_{L,F}\}/B.$

**Appendix B: Experiment 1B**

Our analyses in Experiment 1 showed that area rule of lightness is applicable for relative area values less than 0.5. This is not consistent with the results in the literature (Gilchrist & Radonjić, 2009; Li & Gilchrist, 1999). To test this further, we directly compared the lightness estimates for contiguous sectors with relative areas below 0.5.

**Methods**

The same volunteers participated in a temporal two-alternative forced-choice (2AFC) procedure using...
the same experimental apparatus. Stimuli were 2D images of disks with two radial sectors as in the Experiment 1 contiguous condition. In each trial, two stimuli were presented separated in time. The stimulus duration was 1 s, with an interstimulus interval and intertrial interval of 1 s (after the participant’s response). Participants were asked to indicate the interval in which the dark sector appeared lighter. The standard stimulus had an arc angle of 90° (\(A = 0.25\)) and 8 cd/m² luminance. The comparison stimuli had 45° (\(A = 0.125\)) and 135° (\(A = 0.375\)) arc angles, and the luminance of the dark sector at each trial was adaptively determined by the computer program using a one-up one-down staircase procedure. For each comparison stimulus, two interleaved staircases were used, with 30 repetitions each.

**Results**

We computed the point of subjective equality for the \(A = 0.125\) and \(A = 0.375\) conditions for each observer, as well as the group average and standard deviation. Results are shown in Figure 8. The 45° wedge with \(L = 8.4\) cd/m² is perceived to have same lightness as the 90° wedge with \(L = 8\) cd/m². The 135° wedge with \(L = 7.5\) cd/m² is perceived to have the same lightness as the 90° wedge with \(L = 8\) cd/m². These results suggest that 45° wedge is perceived darker than 90° wedge, and the 90° wedge is perceived darker than the 135° wedge with the same luminance. These results are consistent with our findings in Experiment 1, showing that the area rule of lightness is applicable for relative areas less than 0.5.