The geometric structure of color

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Color is commonly described in terms of the three perceptual attributes—hue, saturation, and brightness—of which only hue has a qualitative nature, saturation and brightness being of a quantitative nature. A possible reason for such a phenomenological structure of the color manifold, and its geometric representation, are discussed.

There has been a long tradition to geometrically represent color (for a comprehensive historical review of various geometrical color models suggested so far, see Kuehni & Schwarz, 2008). The colors of light are commonly represented as a three-dimensional (3-D) cone (Helmholtz, 1867; Schrödinger, 1920/1970; Weinberg, 1976; Wyszecki & Stiles, 1982), whereas the colors of reflecting objects lend themselves to a description as a 3-D ball (Kuehni & Schwarz, 2008; Logvinenko, 2009).

Formally, light color can be defined in terms of color matching. As is well known, color matching can be formalized as an equivalence relation (metamerism) on the set of lights, color being defined as a class of metameric lights (Krantz, 1975). As noted first by Grassmann, metamerism is an equivalence compatible with the linear structure of the light set (Grassmann, 1853; Krantz, 1975). The light set has the structure of a cone in the infinite-dimensional linear space. As a result, a quotient set (with respect to metamerism) of the light cone inherits the conical structure, which allows color to be represented as a convex cone in the 3-D linear space, which is referred to as the color cone (Krantz, 1975).

Alternatively, the color cone can be defined as the image of all the lights in the cone excitation space (Logvinenko, 2014a). Each point in this space represents all the lights invoking the same triplet of cone excitations (Smith & Pokorny, 1996). A triplet of cone excitations will be referred to as a color signal. It is generally believed that lights producing equal color signals appear the same color (Wyszecki & Stiles, 1982). In other words, all the lights mapping to the same point in the color cone make a full (metameric) color match.

Different points in the color cone represent different colors.

Nonmetameric lights appear as having different colors. It should be noted, however, that there are color differences of two types: qualitative and quantitative. Indeed, a difference in hue (e.g., between red and green) is certainly of a qualitative nature (i.e., it is a difference in qualia), whereas a difference in brightness (and/or saturation) is of a quantitative nature (i.e., it is a difference in degree). As matter of fact, unexperienced observers do not seem to count a difference in brightness (and/or saturation) as a difference in color. In other words, for them color means hue.

As color can be defined in terms of metameric (total) color matching of lights, so can hue be defined in terms of hue matching of colors. As a matter of fact, observers are capable of judging whether any two lights of different color have the same or different hue. As known, the monochromatic lights (in the visible part of the spectrum) have different hues. Interestingly, the monochromatic lights of different intensities that match each other in hue do not always have the same wavelengths. This is known as Bezold-Brucke effect (Wyszecki & Stiles, 1982, p. 422). Therefore, hue is not just a perceptual counterpart of wavelength.

Mixtures of two monochromatic lights at the ends of the visible spectrum have so-called nonspectral (purple) hues. A closed loop made by connecting the visible spectrum ends through the (purple) interval is known to exhaust all the hues (Wyszecki & Stiles, 1982). It will be referred to as the hue loop, its image in the color cone being referred to as the hue contour. Any other light is perceived as having either the same hue as one of the lights in this loop (differing perhaps in brightness and/or saturation), or no hue at all. In this latter case the light is perceived as hueless (or neutral). Daylight is usually perceived as neutral. Generally, inside the color cone there is a line (curvilinear ray) coming out of the vertex that represents hueless (neutral) lights.

Formally, hue matching can be described as a map of the interior of the color cone excluding the neutral line onto the hue contour. Given a point on the hue contour its preimage comprises the color signals having
the same hue as it, that is, an equihue subset which makes a two-dimensional (2-D) petal in the color cone. It turns out that the map in question determines a fiber bundle, the equihue subsets being the fibers (Logvinenko & Levin, 2014). They will be referred to as hue fibers. Hence, the 3-D manifold of light colors (i.e., the color cone) is decomposed into (a) a closed one-dimensional (1-D) manifold of hue fibers (which is topologically equivalent to a circle), and (b) the neutral line.

Each hue fiber can be linearly ordered in two different ways: with respect to (a) brightness and (b) saturation. Furthermore, there exists an operational procedure—heterochromatic brightness matching—which allows equating two arbitrary lights in brightness by varying only their intensity (Wyszecki & Stiles, 1982). As a result, the color cone can be sliced into a family of equibright subsets. Every equibright subset can be represented as a closed area topologically equivalent to a disc. The traces of hue fibers in an equibright subset make a fan-like pattern of curvilinear radii (usually referred to as constant hue loci) diverging from a neutral point (i.e., the trace of the neutral line; S. A. Burns, Elsner, Pokorny, & Smith, 1984; Hung & Berns, 1995; Pridmore, 2007). When moving along such a curvilinear radius toward the neutral point, saturation is known to decrease, vanishing at the neutral point.

So, one can see that, firstly, not every light has hue. There are hueless (also called neutral) lights. Secondly, the 3-D light color manifold, or to be more exact, the color cone without the neutral line, retracts to the 1-D hue palette (i.e., the hue fiber bundle). Is there any point in such a retraction?

One can argue that this retraction simply reflects such basic facts that, first, any spectral color can be represented as (i.e., metamer to) a mixture (i.e., weighted sum) of a monochromatic light and the neutral light. Second, the more the daylight weight, the less the saturation of the spectral color. The idea is that the physical mixture of the monochromatic light and the neutral light is experienced as a perceptual (phenomenological) mixture of the hue corresponding to this monochromatic light and the neutral color (corresponding to the neutral light). In other words, as the pure monochromatic light is being physically diluted with the neutral light, the pure hue is being perceived phenomenologically diluted with the neutral color. Furthermore, physically diluting a light, experienced as a pure nonspectral hue (e.g., a mixture of the two monochromatic lights at the opposite ends of the visible spectrum), with the neutral light is similarly perceptually represented as phenomenological dilution of the corresponding nonspectral hue with the neutral color.

Such a physicalist view has been subjected to serious criticism (Mausfeld, 2003). In the present context it is important to note that a similar retraction also takes place for the object colors the stimulus correlates of which are widely believed to be the spectral reflectances of opaque objects. A notion of mixture is not as easy to apply to reflecting objects. Moreover, the neutral (gray) color is found to be no component of any other color (Logvinenko & Beattie, 2011), and this undermines the very basis of the physicalist view in question.

Specifically, the 3-D manifold of object colors, except gray, (i.e., the object-color ball without the center) retracts to the 2-D manifold of so-called optimal colors that are represented by the boundary of the object-color ball (i.e., a 2-D sphere; Logvinenko, 2009, 2013). As known, under a fixed illuminant, the color signals from all the reflecting objects make a closed convex body in the color cone referred to as the object-color solid (Koenderink, 2010; Logvinenko & Levin, 2014; Schrödinger, 1920). It is generally accepted that all the object colors are in one-to-one correspondence with the color signals in the object-color solid. The latter can be smoothly transformed into a 3-D ball so that the spherical coordinates in it correspond reasonably well to the perceptual attributes of object color (Logvinenko, 2009). In particular, when moving from the boundary point of the object-color ball toward its center, one experiences mainly a change in purity, which can be described as the strength of the chromatic quality of the object color. In other words, purity plays the same role for object colors as saturation does for light colors.

Using a new technique—partial hue-matching (Logvinenko, 2012)—it has been confirmed that, in line with the opinion of Da Vinci, Hering, and many others (for a review, see e.g., Valberg, 2001), the objects colors comprise six component (elemental) colors: yellow, blue, red, green, white, and black (Logvinenko, 2012; Logvinenko & Beattie, 2011; Logvinenko & Geithner, in press). Moreover, the six-component structure was found to remain the same irrespective of purity (Logvinenko & Beattie, 2011) and illumination (Logvinenko & Beer, 2012). These studies revealed a special status of gray. As mentioned above, gray was found to be included in no other color (Logvinenko & Beattie, 2011). Thus, gray was found to be essentially different from white and black that were found to be components of many other colors, playing the same role of components as yellow, blue, red, and green. Such a special status of gray leads to the conclusion that it is not a color at all. Objects which appear gray (they map to the object-color solid center) can be thought of as having zero purity.

For every point in the object-color solid apart from gray, one can find another one closer to the object-color solid boundary that differs from it only in
(higher) purity. Therefore, identifying all the colors differing only in purity together, one can retrace the object-color ball (without its center) onto the object-color ball boundary (i.e., a 2-D sphere). Thus, the color palette of object colors is 2-D.

All the points within the object-color (solid) ball (without its center) mapping to the same point in the object-color (solid) ball boundary will be referred to as a hue fiber. In other words, all the object colors differing only in purity belong to the same hue fiber. Such terminology implies that the optimal (boundary) object colors all have different hues. This, in turn, implies that black and white are also hues. Although this is hardly in line with common usage of the these color terms, we will treat black and white as achromatic hues.7

Once again, similar questions arise as above. Firstly, why is the dimensionality of the color manifold reduced to two for object colors and to one for light colors? Secondly, why does the chromatic quality decrease when one approaches the object-color (solid) ball center? Lastly, why is the object-color (solid) ball center colorless?

In attempt to find a plausible answer to these questions, consider first object colors. It is generally taken for granted that object color is a perceptual attribute of an object. In other words, the color of an object remains the same despite a possible change in the viewing conditions and the object’s illumination. In the literature this phenomenon is known as color constancy (Brainard & Radonjic, 2014; Foster, 2011; Jameson & Hurvich, 1989; Katz, 1911/1999; Pokorny, Shevell, & Smith, 1991; Shevell & Kingdom, 2008). While the color-constancy phenomenon is by and large in line with our immediate observation, it poses a serious problem for color science. Indeed, as has been argued before, color constancy is hardly possible for the vast majority of reflecting objects, if any, because of metamer mismatching (Fairchild, 2005; Logvinenko, 2013; Logvinenko, Funt, Mirzaei, & Tokunaga, 2014; Logvinenko & Tokunaga, 2011).

The problem is that when the illumination varies, an object reflects different light invoking different cone excitations under different illuminations. At first glance, if the visual system is capable of making up for the change of the cone excitations produced by an object when its illumination varies, then color constancy can be achieved. Enunciated by Helmholtz (1867), this idea underlies the current computational approaches to the color constancy problem (Ebner, 2007; Gijsenij, Gevers, & Weijer, 2010; Hurlbert, 1998). However, it is impossible to implement because a change in the cone excitations induced by illumination depends on which object invokes those cone excitations. In particular, two objects producing the same cone excitations under one illuminant can produce different cone excitations under some other illuminant. It is this phenomenon that is referred to as metamer mismatching. Obviously, in such a case there is no transformation of the color signal space that can compensate for the illuminant change (Logvinenko, 2013).

True, there is a subset of reflecting objects for which such a compensation for illuminant can be done. These are the objects mapping to the boundary surface of the object-color solid (correspondingly, the object-color ball). Such objects are commonly referred to as optimal reflectances (Wyszecki & Stiles, 1982). As a matter of fact, for each point of the object-color solid boundary there exists just one reflecting object mapping to it (Logvinenko & Levin, 2014). As in this case, there can obviously be no metamer mismatching for optimal reflectances; one can, in principle, implement the Helmholtzian idea for optimal reflectances.

Thus, color constancy can be, in principle, achieved only for the optimal reflectances. For all other reflecting objects, color constancy is, strictly speaking, impossible. In other words, the color of nonoptimal reflectances will undergo unavoidable changes with illumination, that is, they will exhibit inconstancy. Estimation of the range of metamer mismatching shows that the color of an object can vary with illumination on a very large scale (Logvinenko, Funt, & Godau, 2014; Logvinenko, Funt, Mirzaei, & Tokunaga, 2014).

As color is supposed to be an intrinsic feature of an object (Shepard, 1992), there is no point in assigning color to those objects that cannot be color constant. Such objects have to be left without color, that is, to be colorless. Thus, we come to the conclusion that there must one special (improper) “color” that should be assigned to indicate that there is no color for these objects.

In principle, the entire interior of the color solid should get no color, that is, to be colorless. Furthermore, it is worth mentioning that the optimal spectral reflectances are so special that they are highly unlikely to come across in the real world.8 It follows that real reflecting objects would always be experienced as colorless, provided that one sticks to the principle that object color should be absolutely constant with respect to illumination.

However, one can loosen this principle. Indeed, it has been found that the amount of metamer mismatching depends on the position in the object-color solid: the closer to the object-color solid boundary, the lesser metamer mismatching (Logvinenko, Funt, & Godau, 2014; Logvinenko, Funt, Mirzaei, & Tokunaga, 2014; Logvinenko & Levin, 2014). Therefore, the spectral reflectances mapping close to the object-color solid boundary could exhibit approximate color constancy. It seems reasonable to set up a critical value of metamer mismatching, exceeding which will mean that
the improper “color” should be assigned. Hence, the object-color solid boundary with adjacent points (meeting the criterion of mismatching) will get the proper colors whereas the rest of the object-color solid the improper “color.”

The problem with such a line of reasoning is that there is some uncertainty concerning at what level to set up the critical value. On the one hand, it is clear that the larger the metamer mismatching for a given set of the cone excitations, the less reason to assign a proper color, and the more reason to assign the improper one. On the other hand, it is not easy to decide what amount of metamer mismatching to take as the critical value. In other words, the criterion cannot be sharp. Thus, partitioning of the object-color solid into two complementary subsets: the inner part of the object-color solid—which would be colorless—and the outer part—which would be colorful—cannot but be fuzzy rather than sharp.

Such a situation lends itself to using the notion of “fuzzy” subset. As known, a classical (“crisp”) subset $X$ on a set $A$ can be described with a binary (membership) function $\mu_X: A \rightarrow \{0; 1\}$ such that for any element $a$ in $A$ $\mu_X (a) = 1$ if $a$ belongs to this subset, and 0 if the otherwise. Note that the complement $X'$ of the subset $X$ has a membership function $\mu_X' (a) = 1 - \mu_X (a)$. Fuzzy subset is defined with a membership function that is not binary and can take any value in the real interval $[0, 1]$ (Zadeh, 1965). Therefore, the belongingness of a point to a fuzzy subset can gradually vary from 1 (absolutely belongs to 0 (does not belong at all).

The classical notion of fuzzy subset has been generalized by allowing the range of membership function to be an ordered set $^a$ (Goguen, 1967). In other words, the actual values of the membership function do not matter. What matters is in what order they are. In particular, let $\preceq$ be the order on the membership function range, then if elements $a$ and $b$ in $A$ are such that $a \preceq b$ then the grade of membership of $b$ in $A$ is stronger than that of $a$.

It seems natural to suppose that, given two points, $x$ and $y$, in the object-color solid, the belongingness of $x$ to the colorless subset is stronger than that of $y$ if metamer mismatching for $x$ is larger than for $y$. And vice versa, the belongingness of $x$ to the colorful subset is stronger than that of $y$ if metamer mismatching for $x$ is smaller than for $y$.

Such a definition implies some measure of metamer mismatching. As metamer mismatching reveals itself with dispersing the color signals, produced by all the reflectances metamer under one illuminant, into a volume under the other illuminant, it seems plausible to quantify the amount of metamer mismatching by this volume normalized by the object-color solid volume. Such a ratio, varying between 0 and 1, has been proposed as an index of metamer mismatching (Logvinenko, Funt, & Godau, 2014; Logvinenko, Funt, Mirzaei, & Tokunaga, 2014; Logvinenko & Levin, 2014). It was found that this index monotonically decreases when moving from the object-color solid center to the object-color solid boundary, reaching zero at the boundary (Logvinenko, Funt, & Godau, 2014; Logvinenko, Funt, Mirzaei, & Tokunaga, 2014). This index seems to be a reasonable basis for defining the two complementary fuzzy subsets: colorful and colorless ones. Specifically, denoting $i(x)$ the metamer mismatch index of a point $x$ in the object-color solid, the belongingness of $x$ to the colorful subset is supposed to be stronger than that of $y$ if $i(x) < i(y)$.

Furthermore, after normalizing to its maximum, the metamer mismatch index can be considered as the (classical) membership function of the colorless subset. Respectively, one minus the normalized metamer mismatch index will be the (classical) membership function of the colorful subset.

Subjectively, the belongingness to the colorful subset is experienced as the purity of the color, that is, its subjective strength. The fact that the purity of the object colors gradually increases when moving from the object-color solid center, means that the colorless core of the object-color solid has a fuzzy boundary.

It should be noted that without differentiating elements within a hue fiber (that is, if all the elements of a hue fiber were perceived as the same color), there would be a contradictory situation when equally illuminated objects inducing different color signals would be perceived as having the same color, that is, identically. In reality they are perceived of the same color but of different strength (purity). That is, we are well aware of when two objects are of the same color but of different quantitative strength, and when they are of qualitatively different colors.

It is worth mentioning also that gradual decrease in purity toward the object-color solid center makes the latter a limiting point for each hue fiber. In turn, this secures the smoothness of the object-color manifold. Specifically, as pointed out as early as by Grassmann (1853), points close to each other in the color signal space are perceived also subjectively close, that is, they produce a small color difference between them. In the view of the hue fiber bundle over the sphere, such smoothness should not be taken for granted. Indeed, it is known that one cannot continuously map a 3-D ball onto its boundary sphere so that the boundary maps onto itself. At least one point must be left out of the domain of such map for the map to be continuous. By the way, this circumstance alone leads to the necessity of hueless color’s existence. Note that, generally speaking, the hue bundle and the hueless color are disconnected. Purity order, such that the hueless color becomes a limiting point for each hue fiber (in the topology induced by this order), results in the
smoothness and connectivity of the object–color manifold.

The existence of such inner structure of the object color sheds light on the most intriguing aspect of color constancy, which is revealed by the existence of asymmetric object–color matching. It is more or less understood now that color constancy cannot be taken literally, that is, color appearance of objects does not remain constant when their illumination changes (Foster, 2003; Logvinenko & Tokunaga, 2011). Nevertheless, human observers are capable of doing asymmetric object–color matching, that is, they are capable of putting in correspondence the colors of differently illuminated objects. Therefore, there must be some perceptual basis for such matching (Cohen, 2008).

One has to realize that the very existence of asymmetric object–color matching poses a big problem since there is no physical basis for it. Indeed, consider two object–color solids induced by two different illuminants. Asymmetric object–color matching can be formally described as a map of one object–color solid onto the other (referred to as the asymmetric object–color matching map). The only physical basis for such map is the correspondence between those points of the object–color solids that are produced by the same reflecting objects. However, as shown elsewhere, because of metamer mismatching, this correspondence is of a many-to-many type for all the interior points of the object–color solid (Logvinenko, 2013). The degree of metamer mismatching for most points is so large (Logvinenko, Funt, & Godau, 2014; Logvinenko, Funt, Mirzaei, & Tokunaga, 2014) that it makes it impossible even to approximate this correspondence more or less reasonably by any one-to-one map. Thus, the problem is not that it is impossible to arrange a physically plausible asymmetric object–color-matching map (between the two object–color solids) so as to secure color constancy; the problem is that it is impossible to arrange a physically plausible asymmetric object–color-matching map at all (Logvinenko, 2013).

The problem becomes more tractable if one considers the object–color solids “sliced” into hue fibers. Indeed, consider the two hue fiber bundles corresponding to some two different illuminants. It is logical to assume that asymmetric color matching can be formally considered as a (one-to-one) correspondence between these hue fiber bundles. In other words, the asymmetric object–color-matching map reduces to a map of one hue fiber bundle onto the other (referred to as the asymmetric hue-fiber-bundle map). As each hue fiber contains one and only one point belonging to the object–color solid boundary, this correspondence can be specified with a map of the sphere of optimal reflectances onto itself. For color constancy to hold for optimal reflectances, this map should be the identity function, that is, mapping any optimal reflectance on itself (referred to as the optimal asymmetric hue-fiber-bundle map). In other words, the optimal colors will be perceived constant if the asymmetric object–color-matching map puts in correspondence those color signals (in the object–color solid boundaries) that are produced by the same optimal reflectance. Whether this is the case or not can be established only by experiment. Still, there is every indication that even if this is not exactly the case, the asymmetric object–color-matching map is likely to be close to the identity function. Really, white tends to be perceived as white irrespective to illumination. Introspectively, this fact is usually described by observers as that although it does not seem to be exactly white, it looks as if it is white under this chromatic illumination. It should be emphasized that the hue fiber corresponding to the north pole of the object–color solid (and experienced as white) under neutral illumination (e.g., daylight) is put in correspondence to the hue fiber corresponding to the north pole of the object–color solid under chromatic illumination not because it is experienced as the same white, but because they both occupy the same place in the spherical structures of the hue fibers. In other words, both “whites” have the same structural position in the hue fiber bundles.

It is important to note that even the optimal asymmetric hue-fiber-bundle map does not prevent color inconstancy for nonoptimal reflectances. As argued elsewhere, metamer mismatching results in the so-called illuminant-induced color stimulus shift (Logvinenko, 2013). As a result, a reflective object appearing gray (colorless) under one illumination might appear greenish under some other. However, the illuminant-induced color stimulus shift can be computed from the spectral reflectance function and the illuminant spectral power distributions (Logvinenko, 2009). Hence, if the asymmetric hue-fiber-bundle map is known, the color shift caused by the illuminant-induced color stimulus shift can be predicted. Thus, the degree of color inconstancy for any nonoptimal reflecting object can be theoretically estimated.

Hence, the object–color shift invoked by an illuminant alteration can be split into two components: one resulting from the optimal reflectance’s inconstancy (which can be minimized by arranging the asymmetric hue-fiber-bundle map as close to the optimal asymmetric hue-fiber-bundle map as possible), and another (unavoidable) resulting from metamer mismatching. Only the first component has to be determined by experiment. The second one can be derived from the illuminant-induced color stimulus shift (Logvinenko, 2009).

So, we see that the hue fiber bundles are the key to the color constancy problem. They cannot be theoretically deduced. They have to be identified by experiment for both illuminants. Then, the asymmetric hue-
fiber-bundle map has to be identified. It will show which hue fiber under one illuminant corresponds to which hue fiber under the other illuminant. Note that the identification of the asymmetric hue-fiber-bundle map does not require presenting the optimal reflectances. Any physically implementable reflectances representing the hue fibers will do.

Interestingly, the identification of the asymmetric hue-fiber-bundle map essentially simplifies when specified in terms of component (unique) hues. There has been a long tradition to specify hue in terms of unique (component) hues, usually referred to as “hue scaling” (Gordon & Abramov, 1988; Gordon, Abramov, & Chan, 1994). Specifically, there is a general belief that any hue can be decomposed into a (phenomenological) sum of the component hues that appear unique (Hering, 1920/1964; Kaiser & Boynton, 1996; Valberg, 2001). The component hues are included in the sum with some weights,\(^1\) that can be used as coordinates (referred to as the “color coordinates”) of the hue in question. This idea has been implemented in the NCS object-color atlas (Hard & Sivik, 1981). However, this has been done only for neutral illumination. An obvious problem arising for chromatic illuminations is that the existing methods of “hue scaling” are based on verbal definitions of unique (component) hues. However, under chromatic illumination the color appearance of objects becomes rather complicated. For example, a yellow surface under blue light might appear yellow and blue at the same time. Such a phenomenological dualism has been conceptualized in the notions of material hue and lighting hue of an object-color (Tokunaga & Logvinenko, 2010c). Furthermore, it has been argued that such dualism applies to not only hue but other color dimensions as well\(^2\) (Logvinenko & Maloney, 2006; Logvinenko & Tokunaga, 2011; Tokunaga & Logvinenko, 2010a, 2010b, 2010c).

Clearly, this dualism makes it hard to explain to observers what “unique” hues mean.

Fortunately, this problem can be circumvented by using the partial hue-matching technique, which can be considered as a method of hue scaling without hue naming. Using this technique, one can reveal the number of component hues and the unique hues without presupposing how many of them exist for an arbitrary illuminant. As mentioned above, with the partial hue-matching technique, the same six-component object-color structure has been found not only for the neutral but also for chromatic lights (Logvinenko & Beer, 2012). This allows for evaluating the color coordinates for any two illuminants. By definition, all the colors, the color coordinates of which are proportional to each other, will make a hue fiber.

If the first stage—the specifying hue fibers under two illuminants—is accomplished with the partial hue-matching technique, then the second stage—the identification of the asymmetric hue-fiber-bundle map—becomes trivial, not requiring a special experiment. Really, in this case the second stage reduces to realizing which of the six component hues revealed under one illuminant corresponds to which of the six component hues revealed under the other illuminant.

As known from the previous study (Logvinenko & Beer, 2012), this is not a problem at all. In this experiment the observers were presented two identical sets of Munsell chips lit by different (achromatic and chromatic) lights. They were instructed to decide whether two particular chips (one from each set) share any shade of any hue in common. From their responses one derived not only the component hues under each illumination but also which component hue under one illumination corresponded to which hue under the other illumination (Logvinenko & Beer, 2012).

Hence, using the partial hue-matching technique makes it possible to theoretically predict the object-color shift (thus the degree of color inconstancy) for any reflecting object without resorting to experimentation with this particular object, providing that its spectral reflectance function along with the spectral distribution functions of the illuminants are known.

It should be mentioned that although direct computation shows that the illuminant-induced color stimulus shift is often rather large (Logvinenko, 2009; Logvinenko & Tokunaga, 2011), our intuition is that object-color constancy is quite good. There are two factors contributing into this illusion: the inverse relation between metamer mismatch index and purity, and the small number of component hues.

For example, a reflective object appearing gray (colorless) under one illumination might appear greenish under some other. However, because the purity of this shade of green is likely to be small, the overall color difference (induced by the corresponding object-color shift) will be small too. Generally, while colors of low purity can undergo rather considerable shift in hue (e.g., red might turn green; Logvinenko, Funt, Mirzaei, & Tokunaga, 2014), the overall color difference will not be large. On the other hand, object colors of high purity are not subject to large metamer mismatching (thus, color shift). Respectively, they will not be experienced as altering too much with an illuminant change.

The six-component color structure of the object-color palette also contributes to the mitigation of the illuminant-induced color shift. As well known, most of object colors are either binary (comprising two component colors) or ternary (comprising three component colors) (Hering, 1920/1964; Logvinenko, 2012; Logvinenko & Beattie, 2011; Logvinenko & Geithner, in press). An illuminant alteration quite often (but not necessarily always) induces a color shift that will be experienced as a change in the component color weights.

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\(^{1}\) Logvinenko & Tokunaga, 2010a, 2010b, 2010c.

\(^{2}\) Logvinenko & Tokunaga, 2011.
rather than the component colors themselves. For example, a yellow-green color becomes more greenish and less yellowish. Although the color stimulus shift in the color signal space might be considerable, subjectively the difference between these yellow-green colors will not usually be perceived as significant as compared to, say, a shift from pure yellow to yellow-green.

All this leads to an impression (which has become a widely accepted view) that object-color perception is by and large constant with respect to illumination. At any rate, the color inconstancy observed is usually not easy to notice, at least for inexperienced observers.

In terms of light colors, one should note that metamer mismatching occurs for them for the same reasons as for object colors. Although in colorimetry the atmosphere is never taken into account, it is always present in real circumstances. Indeed, the atmosphere per se and other transparent media (such as water, fog, glasses, contact lenses, films, and the like) alter the spectral power distribution of the light coming to the eyes of an observer. The effect of transparent media can be described using the media transmittance spectral function, which formally plays an identical role to object spectral reflectance function. As a result, metamer mismatching for lights is as unavoidable as for reflecting objects. It has been formally described elsewhere (Logvinenko, 2014b; Logvinenko & Levin, 2014). In particular, it has been shown that two broadband lights metameric in the atmosphere with one spectral transmittance cease to be so in an atmosphere of another spectral transmittance. It follows that there is a color shift induced by a change in atmosphere. In other words, most of lights are color inconstant with respect to atmosphere. Specifically, it has been shown that metamer mismatching does not occur only for the case of monochromatic lights (mapping to the color-cone boundary; Logvinenko & Levin, 2014). Applying the same line of reasoning as above, one can show that representing the color cone as the hue bundle (plus the neutral line), with the hue fibers being ordered so that their saturation vanishes when approaching the neutral line, allows for mitigating the atmosphere-induced inconstancy of light colors.

In the end, note that although it has been known for a long time that color has its inner structure, so far it has neither been properly described nor accounted for. Indeed, describing color in terms of three dimensions such as hue, saturation, and brightness leaves out of the scope the enigmatic fact that these dimensions are not on a par with each other, hue being actually used as a synonym of color. Specifically, the light color-cone layers into 2-D hue fibers (plus the neutral line), the hue fiber bundle being topologically equivalent to a 1-D circle. Likewise, the object-color solid layers into 1-D hue fibers (plus the gray), the hue fibers bundle being topologically equivalent to a 2-D sphere. Such a stratified inner structure of the light and object color manifolds is shown to mitigate an unavoidable alteration of, on the one hand, the color of an object when its illumination changes, and, on the other, the color of light under atmospheric changes.

**Keywords: color, fiber bundle, color geometry**

### Acknowledgments

Commercial relationships: none.
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### Footnotes

1. It turns out that to exhaust all the hues, it is sufficient to connect the ends of the so-called effective visible spectrum (which is somewhat shorter), namely, the monochromatic lights with wavelengths of around 410 and 700 nm (Logvinenko, 2014a).
2. In other words, the hue loop is the effective visible spectrum completed by various mixtures of the monochromatic lights that are the ends of the effective visible spectrum.
3. Actually, the hue contour lies on the color-cone boundary.
4. A definition of fiber bundle can be found, for instance, in Hirsch (1976).
5. However, see (Logvinenko, 2013).
6. This technique allows the set of component hues to be ascertained without postulating in advance their number, and without resorting to verbal naming. Being presented with two colors, observers are asked whether these colors share a shade of any hue (or color) in common. Then, from the matrix of their responses the component hues are derived.
7. An alternative would be to use the term “color fiber” or “optimal color fiber,” which would be even more confusing as we take the difference in purity as the difference, though quantitative, in color.
8. The optimal spectral reflectances are found to be discontinuous step-like functions taking only one or zero (Logvinenko & Levin, 2014; Schrödinger, 1920). Usually, they only have one or two discontinuities.
9. To be more exact, it has to be a lattice.
10. It is the so-called “no retraction theorem” (see e.g., Burns & Gidea, 2005, p. 58).
It is assumed here that one deals with multi-illuminant scenes. Sometimes, color constancy observed in such scenes is referred to as simultaneous color constancy (Brainard, 2009).

As a matter of fact, hue retraction takes place under any illuminant. In other words, the hue fibers are spherically structured under all illuminants (i.e., they make a 2-D manifold topologically equivalent to a 2-D sphere).

In other words, it is assumed that if some color from a hue fiber of the first fiber bundle matches some other color in a hue fiber of the second fiber bundle, then any other color in the first hue fiber matches a color in the same (second) hue fiber. It follows that asymmetric color matching induces an (asymmetric color matching) map between the two bundles.

Such an ability to distinguish between the color of material and that of lighting has been conceptualized in terms of material and lighting object-color dimensions (Logvinenko & Maloney, 2006; Logvinenko & Tokunaga, 2011; Tokunaga & Logvinenko, 2010a, 2010b, 2010c).

These weights are nothing more than the purity of the components in the sum.

It is worth emphasizing that the material color dimensions are not subjective counter-parts of some invariant (with respect to illumination) features of an object (Logvinenko, 2013). Likewise, the lighting color is not a subjective representation of illumination independent of the object. Thus, the approach behind these notions is rather different from that of MacLeod (2003) and Mausfeld (2003), who argue that there must be a neural code for the color of the illumination apart from that of the color of object.

References


