Binocular correspondence and the range of fusible horizontal disparities in the central visual field

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Binocular disparities underlie precise stereoscopic depth perception but only over a finite range. At large disparities, objects appear diplopic, and depth perception is degraded. Measurements of the range of horizontal disparities for which single vision is experienced have previously been restricted to the horizontal plane of regard. We extended these mappings, in two experiments, to the upper and lower visual fields and eccentric meridians. In Experiment 1, we measured empirical corresponding points and fusional limits at identical elevations in the median plane for 20 participants. We observed a vertical shear in binocular correspondence consistent with a backward inclined empirical vertical horopter and the fusional range centered upon it. In Experiment 2, we mapped the vertical horopter and fusional limits for a second set of elevations in the median plane and at two additional eccentricities and found a similar pattern of results as in Experiment 1. For 23 of 25 participants in this study, we found that the relationship between measurements of the vertical horopter and fusional range is similar to the established relationship between Panum's fusional range and the horizontal horopter. Our data replicate previous findings that the vertical horopter is inclined top back. We are the first to illustrate that the fusional range of horizontal disparities is approximately centered upon the vertical horopter in the median plane and along eccentric meridians.

Introduction

Stereopsis is the process of recovering the relative depth of objects in a visual scene based on comparisons of the slight positional differences, called disparities, between the left and right eye images. Binocular disparities can be decomposed into horizontal and vertical components, but horizontal disparities are the main source of information used by the binocular system to recover relative depth (see Howard, 2012). The magnitude and sign of horizontal disparities vary systematically with the depth separation and depth order of the objects. For example, if one fixates his or her left index finger at half an arm’s length while holding his or her right index finger adjacent to it, the disparity of the images of the right index finger is close to zero. Holding fixation on the left finger and moving the right index finger closer to the face results in increasing crossed disparities, and moving the right index finger beyond the left index finger results in increasing uncrossed disparities. For disparities up to a moderate magnitude, objects are perceived as single, and a strong impression of depth is experienced. Ogle (1952) referred to this as patent stereopsis. For larger disparities, objects tend to be seen as double and depth perception is less precise (Blakemore, 1970; Ogle, 1952), a state that Ogle referred to as qualitative stereopsis.

This paper is concerned with the distribution of horizontal binocular disparities for which single vision is experienced in the central visual field and the relationship of this distribution to empirical mappings of binocular correspondence.

One mapping of binocular correspondence is the geometric horopter. This model assumes that the eyes are identical globes with longitudes and latitudes analogous to the geographic coordinates of the Earth. Binocular corresponding points have identical locations (i.e., the same longitude and latitude coordinates) in the two eyes. If we wish to map the locations in near space that project to corresponding points in the two eyes under the geometric model, consider the horizontal plane of regard and symmetrical convergence closer than infinity. The intersection of visual lines extending from corresponding points through the optical nodal points in the two eyes trace an arc intersecting the fixation point and the optical nodal points of the two

eyes, commonly referred to as the Vieth-Müller circle after the independent works of Muller (1826) and Vieth (1818). However, Howarth (2011) showed that a more precise definition of this set of loci is the larger arc of a circle containing the fixation point and the optical nodal points of the eyes because locations on the smaller arc between the eyes do not project to corresponding points in the two eyes and are located inside the head. We refer to this set of loci as the geometric horizontal horopter.

A second component of the geometric horopter traces a line intersecting the fixation point perpendicular to the plane containing the geometric horizontal horopter, called the geometric vertical horopter. This set of loci is derived by mapping the intersection of visual lines extended from corresponding points along the vertical meridian, above and below each fovea, through the nodal points of the eyes. Together, the horizontal and vertical sets of loci form the geometric space horopter for symmetric convergence, establishing a set of loci for a given fixation distance that has zero disparity, and single vision should ensue for normally sighted viewers.

The geometric horopter is based on several simplifying assumptions. For example, it is assumed that corresponding points in the two eyes are congruent. For this to hold, several assumptions must be met: (a) The eyes must have the same optical properties, and the retinas must be identical in shape; (b) superimposing the two nodal points of the eyes results in all pairs of corresponding points being congruent; (c) the movements of the eyes are coordinated and there is no fixation error; (d) the optical nodal points and the center of rotation are coincident in both eyes; and (e) convergence eye movements within the horizontal plane of regard are free of cyclorotary eye movements (Howard & Rogers, 2012).

In human binocular systems, all assumptions underlying the geometric horopters are rarely satisfied. Violations of assumptions a, c, d, and e are typically minimized by testing healthy participants with normal binocular vision and employing appropriate experimental controls. Violations of assumption b cannot be corrected, as we discuss below, and so empirical measurements of the horopter based on observers’ responses to specific criteria are required. The nature of the resulting mapping depends on the criterion employed, and so several different empirical horopters exist, each taking a different form depending on the psychophysical criterion used to measure it (for a review of all horopter types, see, for example, Howard & Rogers, 2012; Tyler, 1991).

The present study is concerned with two horopters: (a) The empirical horopter is based on the criterion of identical visual direction in the two eyes. This horopter has two components: the empirical horizontal horopter and the empirical vertical horopter. (b) The fusion horopter is based on the criterion of single vision. Ideally, this horopter also has two components. However, data has only been published for the horizontal component. One goal of the present study is to provide data for the vertical component.

The empirical horizontal horopter, based on the Nontius or minimum motion procedure, is a shallower curve than the geometric horizontal horopter. This is known as the Herring-Hillibrand deviation (Hillibrand, 1929; Ogle, 1964; Schreiber, Hillis, Filippini, Schor, & Banks, 2008) and is generally thought to be a result of a violation of assumption b described above, which says superimposing the two nodal points of the eyes results in all pairs of corresponding points being congruent. Points on the nasal side of the retina are spread out relative to their empirical corresponding points on the temporal side of the contralateral retina. There are also individual differences in these mappings (e.g., Shipley & Rawlings, 1970). The empirical vertical horopter, based on the same criterion, is often said to approximate a line inclined top away from the observer. Again, this deviation from the geometric model is due to a violation of the assumption of congruence between the retinas. In this case, it is accepted that a vertical shear of corresponding points (increasing horizontal offsets between corresponding points with increasing elevation) exists above and below the foveae. However, a straight line, inclined top away, is a simplification of the empirical vertical horopter. For example, Cooper, Burge, and Banks (2011) and Grove, Kaneko, and Ono (2001) report results that indicate that the vertical horopter deviates significantly from a straight line although it is inclined top away. Nevertheless, these results represent a significant departure from the theoretical prediction (see, for example, Cogan, 1979; Cooper et al., 2011; Grove et al., 2001; Helmholtz, 1909/1962; Ledgeway & Rogers, 1999; Nakayama, 1977; Ogle, 1964; Siderov, Harwerth, & Bedell, 1999). As with the empirical horizontal horopter, the empirical vertical horopter measurements vary considerably across individuals (see Cooper et al., 2011).

The range of horizontal disparities between corresponding images for which single vision is experienced is referred to as Panum’s fusional range (Panum, 1858). The fusion horopter is a mapping of Panum’s fusional range across a set of eccentricities in the visual field. Panum’s fusional range can be determined psychophysically, for horizontal disparities, by increasing the disparity of a target until its images just appear unfused and reducing the disparity of an initially unfused target until its images just appear fused. The near and far boundary values of Panum’s fusional range are inferred from the mean of the increasing and decreasing measurements for crossed and uncrossed disparities, respectively.
The fusion horopter has a specific volume because it specifies the range of horizontal disparities from crossed to uncrossed disparities for which single vision is experienced at various eccentricities. There are several mappings of Panum’s fusional range for locations to the left and right of fixation in the horizontal plane of regard. For example, Fischer (1924) measured horizontal disparity fusion limits for eccentricities up to \(\pm 10^\circ\). Ogle (1952, 1964) reported similar measurements for eccentricities up to \(\pm 16^\circ\). These measurements revealed that the range of fusible horizontal disparities was approximately centered on the empirical horizontal horopter* and that the range of fusible disparities increased with eccentricity. Subsequent studies have examined Panum’s fusional range as a function of eccentricity and concur with Ogle (1952, 1964) and his findings (Crone & Leuridan, 1973; Mitchell, 1966; Ogle & Prangen, 1953; Palmer, 1961).

To the best of our knowledge, there are no contemporary papers reporting systematic measurements of the range of fusible horizontal binocular disparities in the upper or lower visual fields. Meissner (1854) reported a set of measurements on himself with vertical rods and noted that the diplopic images of the rods extended above and below the horizontal plane of regard. Interestingly, he noticed that displacing a rod beyond the fixation point in the median plane of the head resulted in diplopic images that crossed at a certain point above the fixation point. This observation is consistent with measurements of the empirical vertical horopter reviewed above. The disparity between the images of the distant vertical rod and empirical corresponding points decreases with elevation as the loci that stimulate empirical corresponding points recede toward the location of the test rod. The point on the test rod at which the empirical vertical horopter intersects it stimulates corresponding points and therefore is seen as single. The points on the rod below that point have uncrossed disparities relative to the empirical vertical horopter, and points above have crossed disparities. The result is diplopic images of the rod that appeared to cross at some elevation above the fixation point. Meissner’s observations suggest that the range of fusible horizontal disparities in the upper and lower visual field is related to the empirical vertical horopter.

Two recent papers reported data consistent with the proposition that the range of fusible horizontal disparities for locations in the upper and lower visual fields is centered on the empirical vertical horopter. Grove and Ono (2012) found that the range of fusible orientation disparities between near vertical dichoptic lines was shifted toward negative orientation disparities in the upper visual field but toward positive orientation disparities in the lower visual field, consistent with a set of loci tracing a line inclined top away. Grove, Finlayson, and Ono (2014) consistently observed higher fusion thresholds for uncrossed horizontal disparity than for crossed disparity in the upper visual field and the opposite pattern in the lower visual field.

In summary, we report the first systematic measurements of the range of fusible horizontal disparities at locations away from the horizontal plane of regard and compare these measurements with empirical corresponding points measured at the same elevations. We measured the empirical vertical horopter in the median plane (Experiment 1) and at eccentric locations (Experiment 2). We measured Panum’s fusional range at the same locations in the median plane (Experiment 1) and eccentric locations (Experiment 2) in the upper and lower visual field. We compared the empirical horopter measurements with the measurements of Panum’s fusional range to determine if the range of fusible horizontal disparities is biased toward uncrossed disparities in the upper visual field and biased toward crossed disparities in the lower visual field such that the midpoint of the fusional range approaches the empirical vertical horopter.

**Experiment 1**

The purpose of this experiment was to measure empirical corresponding points in the two eyes at several elevations in the median plane and infer the inclination of the empirical vertical horopter and then determine the range of fusible horizontal disparities at those identical elevations. We collected two sets of measurements, ascertaining (a) the direction and magnitude of the horizontal shifts required to satisfy the criteria of minimum apparent motion and (b) the magnitude and sign of horizontal disparities for which single vision is experienced.

**Method**

**Participants**

Twenty participants (seven males, 13 females), including one of the authors (AH), participated. All reported normal binocular vision, and all but AH were naive to the aims of the experiment.

**Apparatus**

The experiment was run on a Mac Pro Computer with stimuli presented on two 24-in. Macintosh cinema displays. Stimuli were drawn and scripted using Matlab with Psychophysics Toolbox extensions (Brainard, 1997; Kleiner, Brainard, & Pelli, 2007; Pelli, 1997). The stimulus display screens were carefully adjusted so that they were aligned and parallel to each other. The
alignment procedure is described in detail in Grove et al. (2001). The displays were viewed via two front-silvered mirrors that were oriented at ±45° to the median plane of the head. Rectangular apertures were placed between the mirrors and their display screens, which served to restrict the participants’ field of view to just the display portion of the screen. Participants’ heads were restrained in a chin and forehead rest, which was adjusted for the participants to ensure their pupils were at the same height as the center of the screens. Responses were collected via appropriate keys on a keyboard. The optical distance and convergence distance were both 65 cm, and one pixel subtended 1.46 arcmin.

**Stimuli**

All experiments employed a central fixation stimulus consisting of a central black dot (diameter 7.3 arcmin) surrounded by a larger black annulus (diameter 21.9 arcmin, width 2.9 arcmin). Black vertical Nonius lines (14.6 × 2.9 arcmin) flanked the fixation stimulus. Test stimuli in the empirical horopter and torsion measurements were round black disks (diameter 69 arcmin) to maximize visibility. Test stimuli in the fusion tasks were vertically oriented black ovals (45 × 11 arcmin), and all stimuli were presented against a white background. We chose ovals over circles because Grove et al. (2014) showed that when participants were instructed to use a strict criterion of diplopia to assess fusion, thresholds increased with stimulus size. Based on preliminary investigations, we decided to minimize this influence by using narrow ovals and instructed participants to use the single criterion of absence or presence of diplopia to evaluate fusion. A similar adjustment of the stimulus was not necessary for the horopter and torsion measurements because the minimum motion judgments involved were not affected by stimulus sizes up to 3° in diameter. Indeed, Grove et al. (2001) and Ledgeway and Rogers (1999) employed disks of 3° diameter for their measurements and yielded data similar to studies using much smaller stimuli (i.e., Nakayama, 1977).

**Procedure**

*Measuring the empirical vertical horopter:* To measure the empirical vertical horopter, we used a modified version of the minimum apparent interocular motion paradigm (Grove et al., 2001; Nakayama, 1977). Dichoptic black disks were presented alternately to the two eyes at 0.83 Hz. At this rate, alternately presented and horizontally separated dichoptic images are perceived to jump back and forth in apparent horizontal motion. Observers were instructed to hold their gaze on the fixation stimulus and monitor the vertical Nonius lines to ensure their alignment while adjusting the horizontal positions of the dichoptic images to minimize the apparent interocular horizontal motion. Left and right arrow keys moved the left and right eyes’ disks in equal steps but in opposite directions. Consider an example trial illustrated in Figure 1. A black disk was initially presented for 600 ms to the left eye 29.2 arcmin to the left of the median plane in the upper visual field. After 600 ms, that stimulus disappeared, and immediately after, an identical disk was presented for 600 ms to the right eye 29.2 arcmin to the right of the median plane at the same elevation and disappeared after 600 ms. Repeating this cycle, the observer perceived a single disk apparently jumping back and forth, horizontally. The observer then adjusted the horizontal separation of the dichoptic disks such that the apparent horizontal motion was at a minimum. Once this was achieved, we inferred that the left and right eye stimuli were falling on corresponding points in the two eyes and recorded the x-coordinate of each eye’s image. These measurements were repeated at eight elevations in the median plane (±467.2, ±350.4, ±233.6, and ±116.8 arcmin). The initial horizontal offset of each of the dichoptic disks was fixed at 29.2 arcmin from the median plane, but the sign of the offset was counterbalanced and randomized across trials. Observers completed four trials, two for each sign of initial offset, at each elevation, making a total of 32 randomized trials. The experiment was self-paced, and viewing time was unlimited.

*Measuring horizontal disparity fusion limits in the upper and lower visual fields:* We measured horizontal disparity fusion limits at the same eight elevations as for the empirical vertical horopter measurements (±467.2,
±350.4, ±233.6, and ±116.8 arcmin). For each elevation, observers completed two ascending trials, in which they increased the disparity of an initially fused target until it appeared diplopic, and two descending trials, in which observers reduced the disparity of an initially diplopic (initial disparity: 73 arcmin) target until it appeared fused. Although the task involved increasing or decreasing disparity, observers were allowed to make fine adjustments involving small reversals in cases in which their settings missed the exact point when fusion broke or returned. A warning tone sounded whenever adjustments were made in the direction opposite to what was intended for the trial to discourage large reversals in settings. Uncrossed disparities were signed positive and crossed disparities negative. The experiment was self-paced, and viewing time was unlimited. Observers completed two trials for each direction (ascending and descending) × two disparities (crossed and uncrossed) at the eight elevations for a total of 64 trials completed in random order.

**Measuring eye torsion:** The backward inclination of the vertical horopter could be accounted for by a vertical shear of corresponding points (increasing horizontal offsets with increasing elevation) above and below the foveae such that corresponding meridians are tilted top-templeward in the two eyes. Alternatively, corresponding vertical meridians may be truly vertical in the two eyes but tilt templeward as a result of cyclotorsion eye movements. These control measurements were obtained so that the contribution, if any, of torsional eye movements could be subtracted from the results we obtained in the vertical horopter and fusion measurements. We used a modified version of the minimum shear of corresponding points measurement and the range of fusible horizontal disparities at the different elevations before reporting those data.

**Results and discussion**

Below, we first report the results from the control experiment measuring torsion. We then report how we used the torsion data to remove the influence of cyclotorsion eye movements on the corresponding points measurement and the range of fusible horizontal disparities at the different elevations before reporting those data.

**Torsion results**

We tabulated the mean elevation of the left and right eyes’ images that satisfied the criterion of minimum apparent vertical motion and plotted these values as a function of eccentricity. We next fit a linear function to the data points for each eye using the least squares criterion. The difference in slopes of the two functions gave an index of torsional alignment of the eyes. If the eyes were torsionally aligned such that the left and right horizontal meridians were aligned, the slopes of the fitted lines should be near zero for both eyes. Negative slopes for the right eye’s settings and positive slopes for the left eye’s settings indicated excyclotorsion. Positive slopes for the right eye’s settings and negative slopes for the left eye’s settings indicates incyclotorsion. Cyclovergence angles were calculated as the angle between the linear fits of the left and right eyes’ vertical adjustments. These values are shown for each observer in column three of Table 1. Positive values indicate excyclotorsion and negative values indicate incyclotorsion.

**Empirical vertical horopter**

We determined the mean horizontal adjustments required to minimize apparent horizontal motion at each elevation. We fitted linear functions to the left and right eyes’ data points using the least squares criterion. We determined the Helmholtz shear angles by computing the angle between the left and right eyes’ linear
functions. Results of this analysis are shown for one observer in Figure 2a. Individual uncorrected shear values are reported in column two of Table 1. We determined individual torsion angles by computing the angle between the left and right eyes’ linear functions fitted to the minimum vertical motion measurements as shown in Figure 2b. Following the procedure employed by Cooper et al. (2011), we next removed the influence of cyclotorsion eye movements on these measurements by shifting the data from the two eyes until the fitted linear functions crossed at the origin. These computations are illustrated for an example observer in Figure 2 with the corrected data shown in Figure 2c.

Individual corrected shear values are reported in column four of Table 1. Inspection of the individual data in the table reveals that the sign of the corrected shear angles are consistent with Helmholtz for 17 of the 20 observers. Testing the null hypothesis that the shear angle is zero, the probability of a negative shear angle would be equal to the probability of a positive shear angle (each 0.5). Therefore, the probability of at least 17 positive values out of 20 is significantly different from chance (binomial test, \( p < 0.01 \)). These measurements replicate the previous findings that the empirical vertical horopter is inclined top away (e.g., Nakayama, 1977).

To illustrate the more subtle aspects of individual variability in the vertical horopter measurements, individual data are shown in longitude–longitude coordinates for each of the 20 observers in Figure 3. This allowed us to link linear adjustments on a tangent display to the spherical retinas (Cooper et al., 2011; Schor, Maxwell, & Stevenson, 1994). We calculated the angular horizontal separation of the left and right eyes’ images required to minimize apparent interocular motion. We removed the influence of cyclotorsion eye movements and removed fixation disparities for each observer’s data using the method described above. The overall patterns of results and individual differences are similar to those reported by Cooper et al. (2011).

**Horizontal disparity fusion limits in the upper and lower visual fields**

Theoretically, a line from the nodal point of the right eye through its image on the display and another line from the nodal point of the left eye through its respective image on the display would intersect at the actual location of a real point in space that would project those images onto the two retinas. Therefore, the horizontal

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<table>
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<tr>
<th>Participant</th>
<th>Uncorrected Helmholtz shear</th>
<th>Torsion</th>
<th>Corrected Helmholtz shear</th>
<th>Corrected midfusional line angle</th>
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**Table 1.** Individual uncorrected shear angles, torsion angles, corrected shear angles (after cyclotorsion eye movements have been removed) for corresponding points and corrected shear angles for midfusional lines. *Note:* Order of participants matches Figures 3 and 4.
adjustments required to eliminate apparent interocular motion are equivalent to the disparity of the real location in space, relative to the plane of fixation that is perceived in the identical direction from both eyes. Owing to this equivalence, we can plot the range of fusible horizontal disparities on the same axes as the empirical horopter data. This allows for a visual comparison between the set of horizontal adjustments required to minimize apparent interocular motion, corresponding to the empirical vertical horopter, and the overall pattern of bias in the range of fusible horizontal disparities as a function of elevation.

Prior to plotting the range of fusible horizontal disparities, we first removed the influence of cyclo-torsion eye movements on fusion thresholds. For individuals exhibiting excyclotorsion, we subtracted the angular contribution predicted at each eccentricity by the least squares line fitted to the cyclo-torsion data from each corresponding disparity fusion threshold. For example, we subtracted the predicted absolute vertical angular separation required to minimize apparent vertical interocular motion in the torsion measurements at an eccentricity of 467.2 arcmin from the uncrossed and crossed fusion limits at the elevation of 462.7 arcmin. The net result across all elevations was to tilt the fusional volume toward vertical in proportion to the measured cyclo-torsion. A similar process was employed for incyclotorsion except that the absolute vertical angular separations measured in the torsion experiment were added to the uncrossed and crossed fusion limits. The net effect across all elevations was to tilt the fusional volume backward in proportion to the measured cyclo-torsion.

Individual horizontal angular adjustments to minimize apparent motion (empirical vertical horopter) and horizontal disparity fusion limits are shown in Figure 4, also in longitude–longitude coordinates. Inspection of the 20 graphs in the figure reveals a consistent pattern in which the range of fusible horizontal disparities is approximately symmetrical about the empirical vertical horopter, indicated by the solid black line, and is similarly inclined backward. Red circles indicate the fusion threshold for uncrossed disparities at each elevation, and green squares indicate those for crossed disparities. The midpoint of this range is indicated by a dotted gray line. To quantify the relationship between the empirical vertical horopter and the midfusional line, we divided the disparity of the midfusional line evenly between the two eyes, fit a line to each eye’s data points using the least squares criterion, and calculated the angle between the left and right eyes’ fitted lines as we did for the corresponding data points above. The resulting angle corresponds to the Helmholtz shear angle for the midfusional line. These angles are reported for all 20 observers in column five of Table 1. Inspection of that column reveals that the Helmholtz shear angles for corresponding points and for the midfusional lines are the same sign for 18 of 20 of the participants. Testing the null hypothesis that the Helmholtz shear angles for corresponding points and for the midfusional lines are unrelated, the probability that the signs of the two measures are equal is 0.5, and the probability that they are not equal is 0.5. Therefore, the probability of at least 18 “like sign” observations out of 20 is significantly different from chance (binomial test, $p < 0.01$).
conclude, therefore, that Panum’s fusional range is biased toward the empirical vertical horopter.

This experiment is the first report of systematic measurements of the range of fusible horizontal disparities for elevations above and below the horizontal plane of regard in the medial plane of the head. Here, we clearly demonstrate that the range of fusible horizontal disparities is approximately centered on the empirical vertical horopter for 18 of 20 participants. In Experiment 2, we further extend these measurements to loci away from the major meridians.

**Figure 3.** Individual plots, in longitude–longitude coordinates, of the angular horizontal adjustments required to minimize apparent interocular motion at elevations in the median plane. Red circles indicate the position of the right eye’s image, and green squares indicate the position of the left eye’s image. Fixation disparities and the influence of cyclotorsion have been removed from each participant’s data. Error bars indicate ±1 SEM. See text for more details.

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**Experiment 2**

This experiment repeated the measurements of corresponding points and the range of fusible horizontal disparities for the vertical meridian in the median plane of the head and two additional eccentric vertical meridians at ±2.9°. In addition, to further link our measurements with previous work, we also measured the horizontal horopter and the horizontal disparity fusion limits around that set of loci, partially...
replicating previous investigations (e.g., Crone & Leuridan, 1973; Ogle, 1952; Palmer, 1961).

**Method**

**Participants**

Five participants, including the two authors and three volunteers, naïve to the purpose of the experiments, participated (two males, three females). All were experienced in psychophysical experiments with normal binocular vision. Four participants completed the entire experiment; one participant (NF) did not complete the measurements along the eccentric vertical meridians.

**Apparatus and stimuli**

The apparatus and stimuli were identical to Experiment 1. The only difference was the choice of eccentricities and elevations at which measurements...
were taken. These differences are noted in the following sections.

Procedure

Measuring the empirical horizontal horopter: The first task of this experiment was to replicate measurements of the empirical horizontal horopter and corresponding measurements of the range of fusible horizontal disparities (e.g., Ogle, 1952, 1964). We collected measurements at each of eight eccentricities (±467.2, ±350.4, ±233.6, and ±116.8 arcmin) centered on the median plane in the horizontal plane of regard. At each eccentricity, the participant’s task was identical to the minimum apparent motion procedure used to measure the empirical vertical horopter in Experiment 1. At a given eccentricity, the left and right eye dichoptic disks were initially displaced to the left and right (or vice versa), respectively, from the objective location to be measured. Observers completed at least four settings to null apparent interocular motion at each eccentricity for a total of 32 randomly ordered trials. Observers with less stable settings completed more. The magnitudes of the initial offsets of the targets and the methods of counterbalancing and randomization were identical to Experiment 1.

Measuring horizontal disparity fusion limits to the left and right of fixation: In the second task of this experiment, we measured horizontal disparity fusion thresholds at the same eight eccentricities (±467.2, ±350.4, ±233.6, and ±116.8 arcmin) as the horopter measurements above. As in Experiment 1, participants completed eight adjustments at each eccentricity consisting of two ascending trials and two descending trials for crossed disparity and uncrossed disparity for a total of 64 trials.

Measuring the empirical vertical horopter in the central visual field: In the third task of this experiment, we measured the empirical vertical horopter at three different eccentricities (0°, ±2.9°). We collected measurements at each of 14 elevations, seven above and seven below fixation in the median plane (±467.2, ±350.4, ±292, ±233.6, ±175.2, ±116.8, and ±58.4 arcmin). Using the same methods as Experiment 1, we collected measurements at these elevations plus measurements at zero elevation at the two eccentric locations (±2.9°). Observers completed at least four adjustments at each elevation. For the eccentric meridians, three observers completed six adjustments, owing to the increased difficulty of the task (PG completed four). The eccentricity of the vertical meridian being tested was blocked. All observers completed measurements along the meridian in the median plane first. Measurements at the two eccentric meridians were collected subsequently and in different orders for each participant. The elevations and initial horizontal offsets between the dichoptic targets were randomized within each block. Observers AH, BE, and MK completed 236 trials (90 at ±2.9° eccentricity and 56 in the median plane). PG completed 56 trials in the median plane and 60 trials at each of the eccentric meridians because his data were less variable.

Measuring horizontal disparity fusion limits in the central visual field: In the fourth task of this experiment, we measured the horizontal disparity fusion limits at the identical locations mapping the empirical vertical horopter. The same stimuli and procedure from the fusion task of Experiment 1 were adapted for this task. Measurements were taken at 14 elevations in the median plane and at 15 elevations at ±2.9°. Participants completed at least two ascending and two descending trials for both uncrossed and crossed disparities for a total of 112 randomly ordered trials in the median plane. At ±2.9° eccentricities, participants completed a similar set of trials totaling 180 trials at each eccentricity. Owing to the difficulty of the task, participants completed additional ascending and descending trials on the eccentric meridians. As in the vertical horopter measurements, data was collected for locations in the median plane first and then along the eccentric locations at a later date. Elevation and initial disparity of the target were randomized within each block. For each elevation, there were a total of 12 observations, three ascending and three descending trials, for crossed and uncrossed disparity. Observer NF completed all measurements in the median plane but was unavailable for testing along the eccentric meridians.

Measuring eye torsion: The fifth and final task in this experiment was to measure the torsional state of the eyes when viewing the displays. As in Experiment 1, these measurements were taken to remove any influence of cyclotorsion on the empirical vertical horopter and horizontal disparity fusion limit measurements. The stimuli and procedure were identical to Experiment 1. Participants completed four adjustments at 14 eccentricities, equal in magnitude to the elevations used in the vertical horopter and fusion tasks (±467.2, ±350.4, ±292, ±233.6, ±175.2, ±116.8, and ±58.4 arcmin) for 56 trials that were randomly ordered for each participant.

Results and discussion

Empirical horizontal horopter

At each of the eight eccentricities, we calculated the mean angular horizontal adjustments required to minimize apparent horizontal motion. These data are illustrated for each of the five observers in Figure 5. Our computations were the same as those for the empirical vertical horopter measurements in Experiment 1. Displacements to the left of the objective point
of interest in the left eye combined with displacements to the right in the right eye were signed positive. The opposite pattern was signed negative. Inspecting the figure, four of the five observers’ settings trace an arc (solid black line) beyond the geometric horopter (horizontal line intersecting the $y$-axis at zero). Close inspection of the individual data reveals some variation in the match between the geometric horizontal horopter and the empirical horizontal horopter. For example, AH’s settings show a close match to the geometric horopter. The remaining four observers’ settings are consistent with uncrossed disparities relative to the geometric horopter as would be expected from previous findings (e.g., Ogle, 1964). H-index values based on the formula

$$ H = \cot(x_L) - \cot(x_R) $$

were calculated at each eccentricity, where $x_L$ and $x_R$ are the angular subtenses of the left and right eyes’ images at the point of minimum apparent motion. Summary values using Ogle’s method of averaging across all eccentricities are shown in column two of Table 2.

**Horizontal disparity fusion limits around the empirical horizontal horopter**

We plotted the range of fusible horizontal disparities on the same axes as the geometric and empirical horizontal horopter data in Figure 5. Inspection of the figure reveals a consistent pattern in which the ranges of fusible horizontal disparities are approximately symmetrical about the empirical horizontal horopter. The dashed gray line indicates the midpoint of the average crossed and uncrossed fusion for each elevation. The midpoint of the fusional range matches the empirical horizontal horopter at all eccentricities for four of the five observers. There is some discrepancy between the midpoint of the fusional range and the horopter for MK owing to larger diplopia thresholds for uncrossed than for crossed disparities. We also computed H-index values for the midfusional line in the horizontal plane of regard using Equation 2. In this case, $x_L$ and $x_R$ are the angular subtenses of the midfusional point at each eccentricity. Summary values using Ogle’s method of averaging across all eccentricities are shown in column two of Table 2.

These data replicate previous findings on Panum’s fusional area, demonstrating that the empirical horizontal horopter is generally a shallower arc than the geometric horizontal horopter and that Panum’s fusional area straddles the empirical horizontal horopter. The small differences between the empirical horizontal horopter and the geometric horizontal horopter in our data preclude us from concluding which horopter lies at the center of the fusional range. Indeed, Ogle (1952) deemed the difference between the geometric and empirical horopters to be of little consequence at similar eccentricities.

**Torsion results**

As in Experiment 1, we tabulated the mean elevation of the left and right eyes’ images that satisfied the criterion of minimum apparent vertical motion and plotted these values as a function of eccentricity. We next fit a linear function to the data points for each eye using the least squares criterion and took the angle
between the two functions as an index of torsional alignment of the eyes. These values are shown for each observer in column four of Table 2. These data were then used to remove the influence of cyclotorsion from the empirical vertical horopter measurements and fusion measurements using the same methods as Experiment 1.

**Empirical vertical horopter in the central visual field**

Using the same computations as in Experiment 1, we determined the mean horizontal adjustments required to minimize apparent horizontal motion at each of the 14 elevations in the median plane and 15 elevations at each of the eccentric meridians. The corrected data, with fixation disparity and the contributions from cyclotorsion eye movements removed, are illustrated in angular terms for each of the five observers in Figure 6. Inspecting the middle column of Figure 6, it is clear that all observers show a clear pattern of adjustments consistent with the backward inclination of the vertical horopter as indicated by the solid black line. Testing the null hypothesis that the shear angle is zero, the probability of a negative shear angle would be equal to the probability of a positive shear angle (each 0.5). Therefore, the probability of five positive values out of five is significantly different from chance (binomial test, \( p < 0.05 \)).

As in Experiment 1, the data show some individual differences in the inclination of the vertical horopter. The left column of Figure 6 displays the mean horizontal adjustments to minimize apparent horizontal motion at each of the 15 elevations \( 2.9^\circ \) to the left of the display; the right column shows the corresponding adjustments for locations to the right of the midline. Inspection of this data reveals a similar pattern to the vertical horopter measured in the median plane. Grove et al. (2001) and Schreiber et al. (2008) reported a similar pattern for empirical vertical horopters at eccentric locations. Individual corrected Helmholtz shear angles for corresponding meridians at each eccentricity are reported in columns five, seven, and nine in Table 2.

**Horizontal disparity fusion limits around the empirical vertical horopter in the central visual field**

As in Experiment 1, we plotted the range of fusible horizontal disparities along with the empirical vertical horopter data in Figure 7. These data have been corrected for the contribution of cyclotorsion eye movements. Inspection of the middle column of the figure shows a consistent pattern in all five observers, in which the range of fusible horizontal disparities is approximately symmetrical about the empirical vertical horopter and matching the backward inclination of the empirical vertical horopter. The correspondence between the empirical vertical horopter and the range of fusible horizontal disparities is clearly illustrated by the close match between the midpoint of the uncrossed and crossed fusion at each elevation (dashed gray line) and the vertical horopter for each of the observers. Testing the null hypothesis that the Helmholtz shear angles for corresponding points and the midfusional lines are unrelated, as we did in Experiment 1, the probability of five “like sign” observations out of five is significantly different from chance (binomial test, \( p < 0.05 \)).

Inspection of the outer two columns of Figure 7 demonstrates that the fusional range is centered on the vertical horopter, following the backward inclination of the vertical horopter at these eccentricities. This is the same pattern seen in the fusion measurements taken in the median plane and is clearly illustrated by the close match between the midpoint of fusional range (dashed gray line) and the empirical vertical horopter settings (solid black line). Individual corrected Helmholtz shear angles for the midfusional line at each eccentricity are reported in columns six, eight, and ten in Table 2.
In summary, we clearly demonstrate that binocular correspondence is sheared for elevations above and below the fovea in the median plane and for positive and negative elevations along the eccentric meridians. Moreover, we have demonstrated for the first time that the range of fusible horizontal disparities follows a similar pattern to binocular correspondence such that it is approximately centered on the empirical vertical horopter in the median plane and along eccentric meridians. Our control measurements of torsional eye movements rule out any significant contribution of these eye movements to the pattern of data collected in the main experiment.

**General discussion**

In this report, we replicated previous research indicating that the empirical vertical horopter is
inclined top away from the observer. Additionally, we conducted the first mapping of Panum’s fusional range for locations in the upper and lower visual fields in both the median plane and at eccentric locations. Comparison of the vertical horopter and fusional volume mappings revealed a clear relationship, both in the median plane and at eccentric meridians, that the empirical vertical horopter has a special status in that it sits in the middle of the range of fusible horizontal disparities. This relationship is the same as the well-established relationship between Panum’s fusional range and the empirical horizontal horopter.

Ogle (1952) measured stereoacuity and Panum’s fusional range around the geometric horizontal horopter and demonstrated that the range of fusible horizontal disparities (figure 4, p. 258) is approximately symmetrical about the horopter. Our extension of Ogle’s measurements reveal a qualitatively similar pattern. Generally, we found fusion limits were centered on the empirical horizontal and vertical

Figure 7. Individual data plots of mean angular horizontal adjustments required to null apparent interocular motion (solid black lines with no symbols) and stimulate empirical corresponding points and the mean fusional limits for crossed disparities (blue squares) and uncrossed disparities (orange circles) as a function of elevation. Dashed gray line indicates the midpoint between fusional limits for crossed and uncrossed disparities. Data have been corrected for the effects of cyclotorsion. Note the x-axis has been expanded relative to the y-axis. Error bars indicate ±1 SEM.
horopters. Moreover, this pattern persisted along the peripheral meridians we investigated in Experiment 2 although the backward inclination of both the empirical vertical horopter and the fusional range were reduced. Ogle also measured the maximum disparity for which an object could be even qualitatively localized in space. For disparities beyond those values, the perception of depth faded, and depth could not be specified. Ogle argued that binocular interaction ceased to occur beyond these large disparities. Experiments are underway in which we are measuring the range of disparities for which patent stereopsis and qualitative stereopsis are supported and the upper threshold at which depth from stereopsis fades across the central binocular field.

Although the present paper is the first to map the range of fusible horizontal disparities around near vertical meridians in the median plain and at two eccentricities, a few studies have investigated the possibility of a horopter surface, extending the set of loci from the primary horizontal and vertical meridians. Ledgeway and Rogers (1999) used a minimum motion technique in which dichoptic pairs of dots, one dot in the upper visual field and the other dot in the lower visual field with the left eye’s dots displaced in opposite directions in the upper and lower visual fields and the right eye’s dots displaced in the opposite direction to the left eye’s. The resulting perception was an apparent change in orientation of the imaginary line joining the upper and lower dots. The observers’ task was to adjust the horizontal offsets of the upper and lower dots to minimize the apparent motion. Their data are consistent with a vertical shear of corresponding vertical meridians in the two eyes. However, because the locations between the dots in the upper and lower visual fields were not explicitly investigated, the vertical shear could only be inferred from their data. Grove et al. (2001) mapped over 50 locations in the central binocular field that fell on empirical corresponding points in the two eyes using the minimum motion technique and reported data consistent with a backward inclined surface that stimulated empirical corresponding points. Schreiber, Tweed, and Schor (2006) modeled various extended horopter surfaces with the concession that points on these surfaces would not project strictly to mathematically corresponding points. However, they predicted that their model surfaces would fall within Panum’s fusional range. Schreiber et al. (2008) later measured empirical corresponding points over the central binocular field, reporting data consistent with both Grove et al. (2001) and Schreiber et al. (2006). The data presented here further support and add to the previous findings, showing that the range of fusible horizontal disparities is approximately centered on these previously reported zero disparity surfaces.

The data from this study can be used to inform developments in 3-D media delivery and presentation. In the applied literature, diplopia is regarded as an undesirable artifact that causes discomfort when viewing 3-D displays, and measures are sought to reduce its occurrence. One solution to reduce diplopia due to excessive disparities in 3-D media is to simply restrict the range of binocular disparities employed to simulate depth (Mendiburu, 2009). Another solution is to tailor the distribution of parallax across the display to maximize comfort. Nojiri, Yamanoue, Ide, Yano, and Okana (2006) investigated the relationship between visual comfort and the parallax distribution across 3-D displays and found that stereoscopic images were rated as more comfortable to view when the lower visual field contained mostly crossed disparities relative to the plane of the display and the upper visual field contained uncrossed disparities. This pattern is consistent with the backward inclination of the empirical vertical horopter. Our novel data mapping a fusional volume around the empirical vertical horopter provides a mapping based on fundamental fusion processes, which is at a more basic level of perception than the subjective ratings used by Nojiri et al.

It is common for 3-D media creators to use the display screen as the zero disparity reference for the distribution of disparities, which approximates the geometric vertical horopter. As we have reported above, the range of fusible horizontal disparities in the upper and lower visual fields are centered upon the empirical vertical horopter, and so it would be useful for those creating 3-D media to use the empirical horopter as their reference point for the distribution of disparities.

Using the data from Experiment 2, we can calculate the average disparities between a frontoparallel plane and the midpoint of the measured fusional ranges. This indicates that if 3-D media creators were to bias their distribution of disparities such that they were centered on the backward inclined empirical vertical horopter they would have an extra 23 arcmin of uncrossed disparities in the upper visual field and an extra 18 arcmin of crossed disparity in the lower visual field at their disposal with a low probability of diplopia. We arrived at this figure by taking the disparity between the frontoparallel display at 65 cm and the midpoint of the fusional range at the top elevation in our experiment. This gain will likely be larger in the context of cinema displays and large-screen televisions, which can accommodate much larger elevations than we explored in this study.

These benefits of using the empirical vertical horopter are more modest along the eccentric meridians; at the −2.9 arcmin eccentricity, both the upper and lower visual fields gain an average of 10.4 arcmin at the furthest measured points. At the 2.9 arcmin...
eccentricity, on average, 14 arcmin of uncrossed disparity at the highest of our measurements in the upper visual field was gained but only 4.25 arcmin of crossed disparity was gained at the lowest of our measurements in the lower visual field. The gains for these eccentric meridians are not as great as those seen in the median plane; however, they still provide a greater advantage to 3-D media developers over the use of a vertical display as a zero reference point. Closer inspection of the individual graphs show that of the five participants in Experiment 2, three (AH, PG, and NF) showed significant backward inclination of the empirical vertical horopter whereas two (BE and MK) showed more modest inclinations. These graphs highlight that research of this nature is often marked by individual differences and that these differences in the inclination of the horopter should correlate with diplopia thresholds for each person as illustrated in Figure 7.

Textbooks and review articles routinely include illustrations depicting Panum’s fusional range centered on both the empirical horizontal and vertical horopters. For example, figure 14.23 (p. 173) in Howard and Rogers (2012) illustrates the geometric horopters, the empirical horopters, and the putative fusional volume around the empirical horizontal and vertical horopters. The relationship between the empirical horizontal horopter and the range of fusible horizontal disparities is well established. Therefore, it is not unreasonable to assume a similar relationship between the range of fusible horizontal disparities and the empirical vertical horopter (see Tyler, 1991, for a similar illustration; see also figures 4–8, p. 90, in Steinman, Steinman, & Garzia, 2000, although they have shown the fusional range around the geometric horopters). Nevertheless, the present report is the first to provide systematic measurements corroborating the presumed link between loci stimulating corresponding points in the two eyes and the range of fusible horizontal disparities.

**Keywords:** Panum’s fusional range, vertical horopter, horizontal horopter, fusion, stereopsis

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**Footnote**

1 Ogle (1952) did not distinguish between the geometric and empirical horizontal horopters in this context.

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